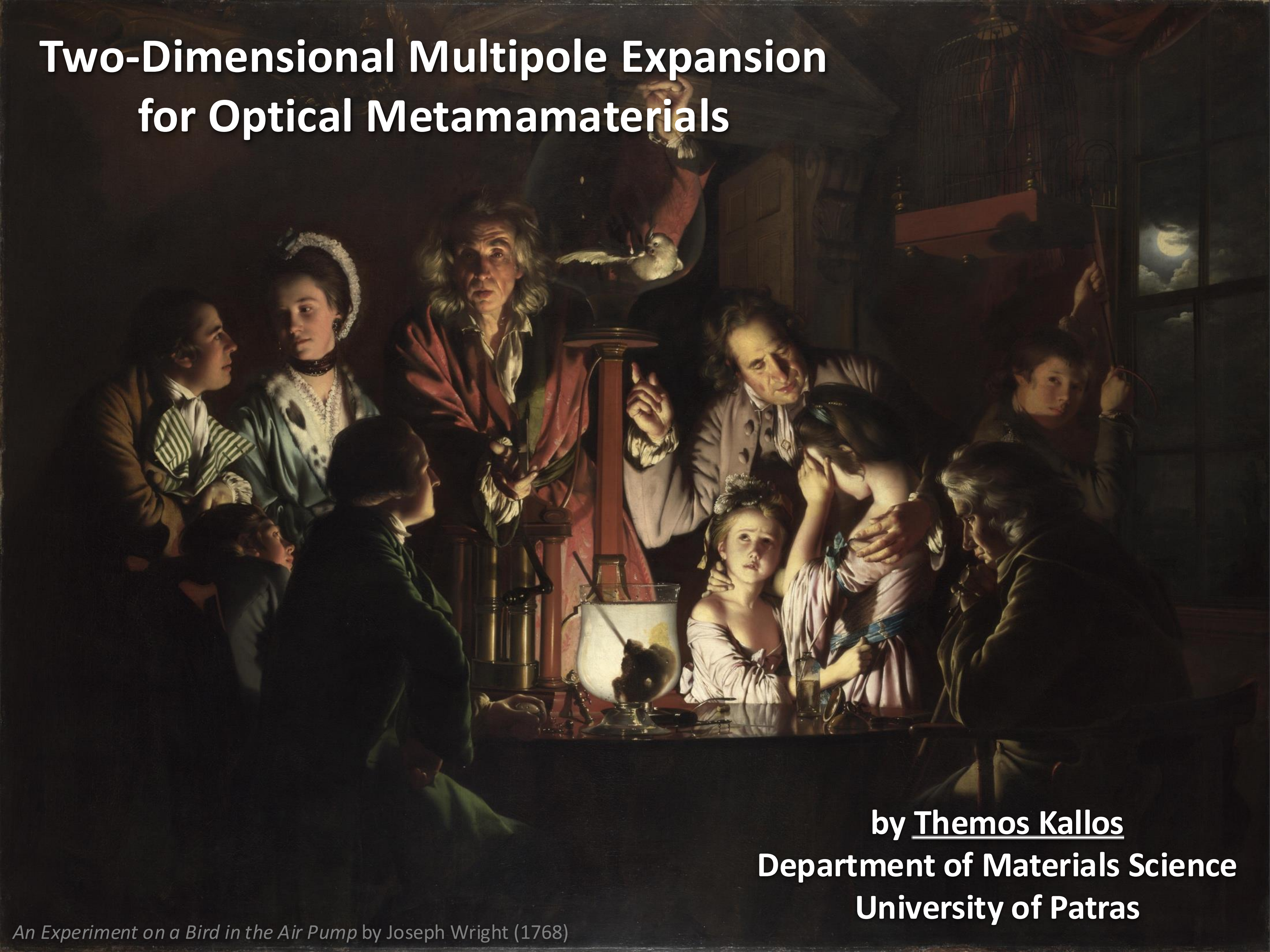
Two-Dimensional Multipole Expansion for Optical Metamamaterials



by Themos Kallos
Department of Materials Science
University of Patras

An Experiment on a Bird in the Air Pump by Joseph Wright (1768)

University of Patras

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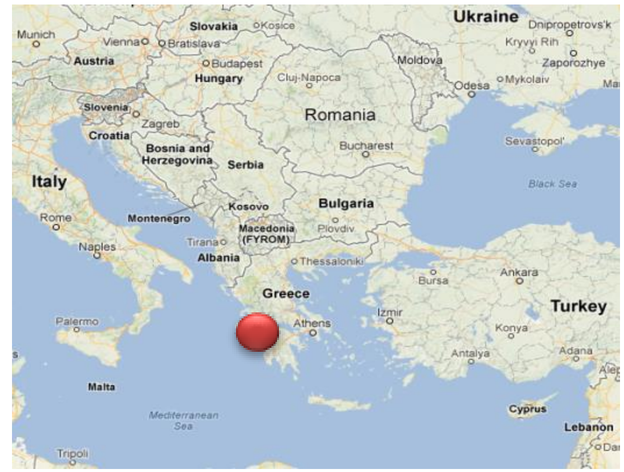
ΠΑΤΡΩΝ
UNIVERSITY OF PATRAS



Acknowledgements

ΠΑΝΕΠΙΣΤΗΜΙΟ ΠΑΤΡΩΝ
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University of Crete

LZH
LASER ZENTRUM HANNOVER e.V.

EPFL
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European Union
European Social Fund

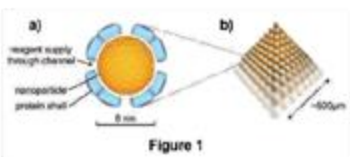
OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION, LIFELONG LEARNING AND RELIGIOUS AFFAIRS
MANAGING AUTHORITY

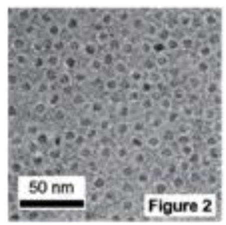
NSRF 2007-2013
programme for development
EUROPEAN SOCIAL FUND

Co- financed by Greece and the European Union

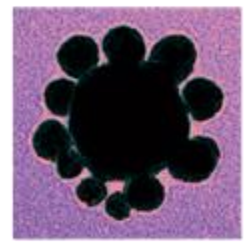
Nanostructured Metamaterials

MAGNONICS – Magnetic Nanostructures
University of Exeter, UK



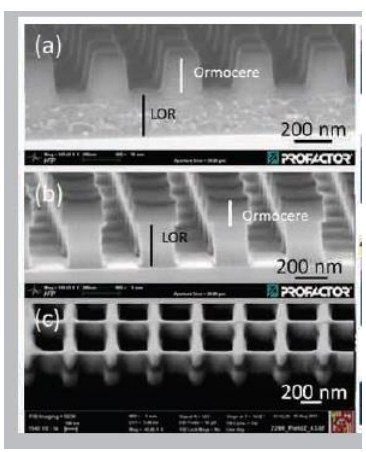


METACHEM – Nanochemictry & Plasmonic Nanoclusters
CNRS-Bordeaux, FR

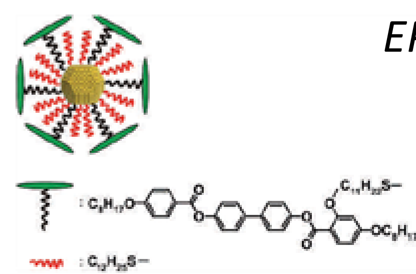


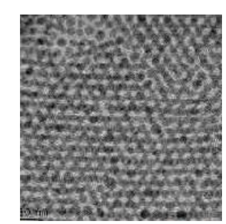
Transmission electron microscopy image of a central 60 nm Au nanoparticle surrounded by 15 nm Au nanoparticles.

NIM-NIL– Metallic Nanostructures using lithography
PROFACTOR GmbH, AT

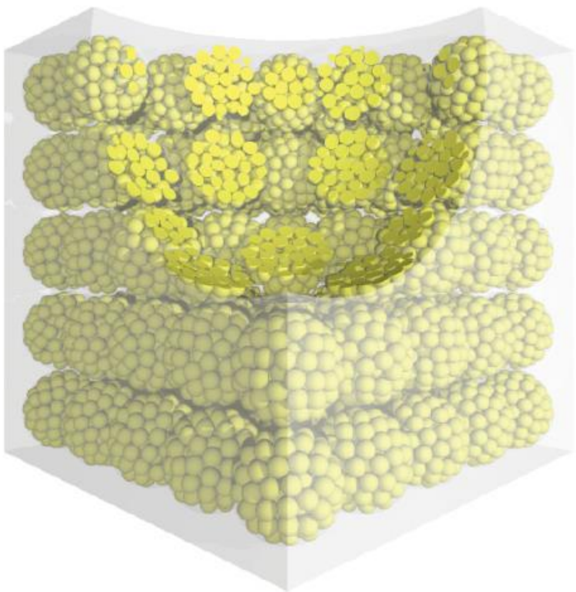


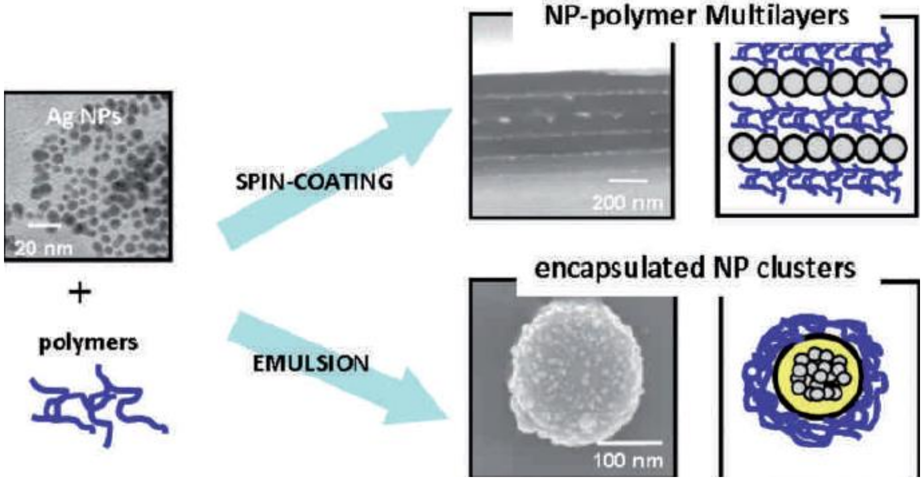
NANOGOLD – Self-assembling nanoparticles
EPFL, CH





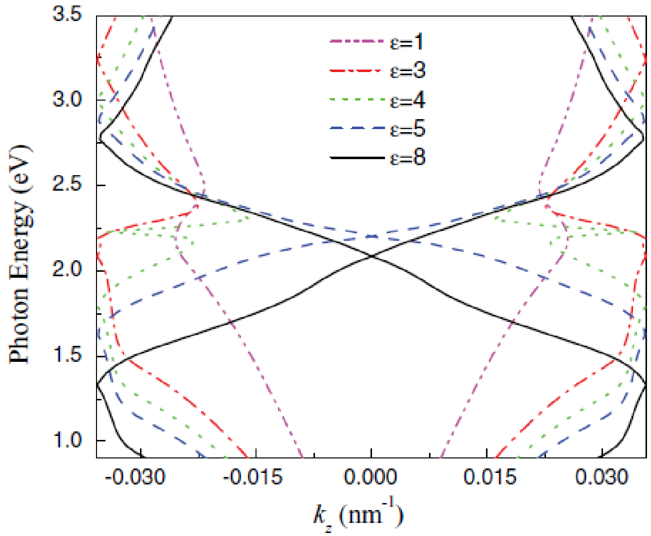
Gold Nanoclusters



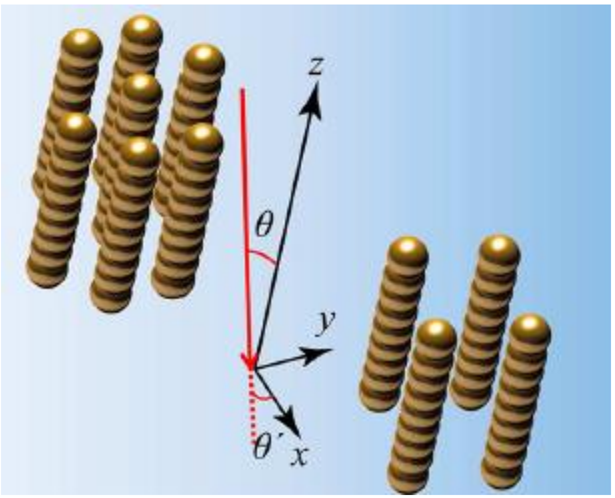


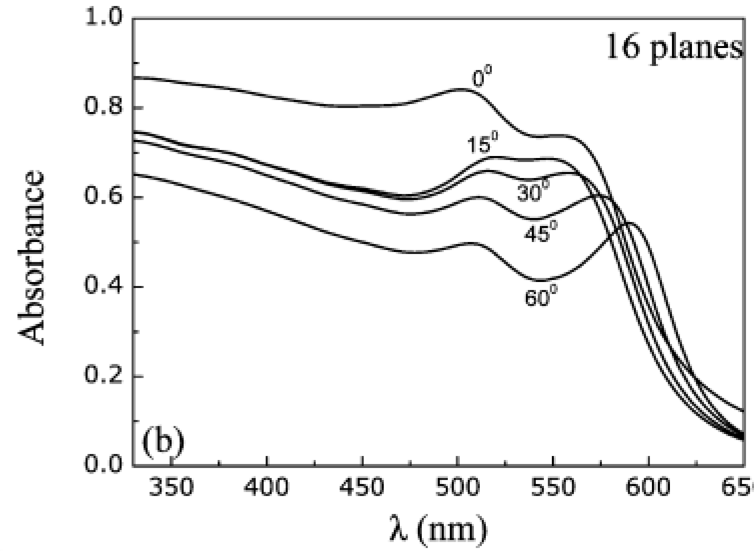
- Gold Nanoparticle radius 9 nm
- Cluster radius 43 nm

- Negative Index meta-metamaterial
- Dirac point



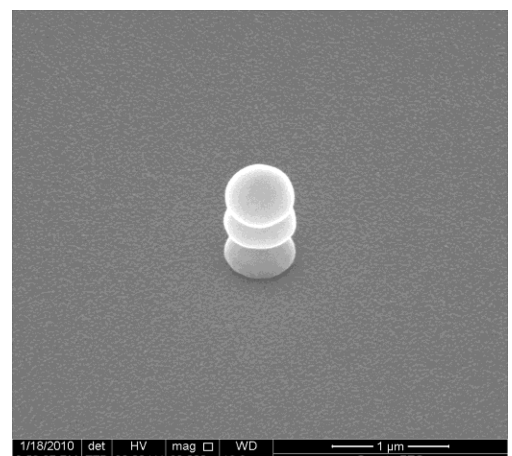
Nanostring Super-absorbers





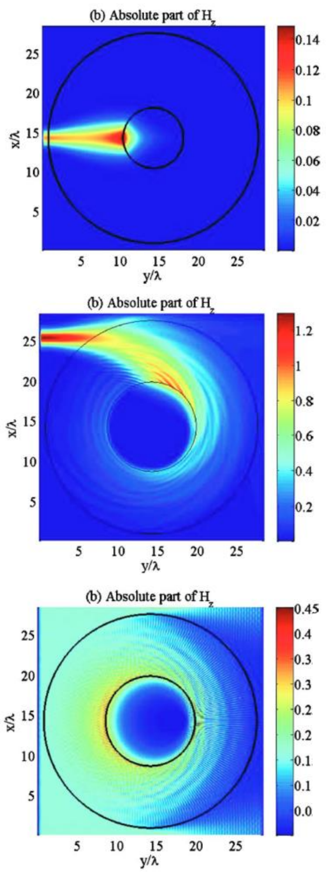
- Hexagonal lattice of gold nanostrings
- Period 10 nm
- Embedded in nematic liquid crystal
- 3 nm diameter

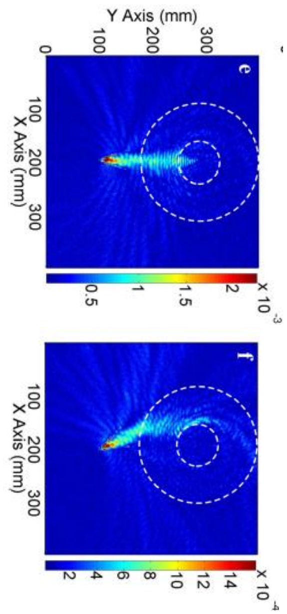
- Gray body: 79% absorption over all angles and polarizations



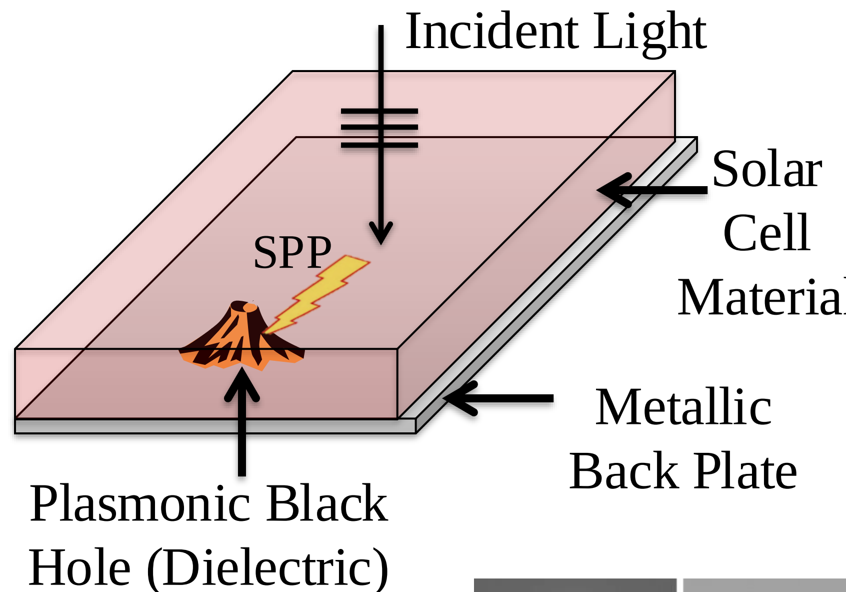
Yannopapas et al., JPCC (2012); SEM Photo by Laser Zentrum Hannover (LZH)

Plasmonic Black Holes

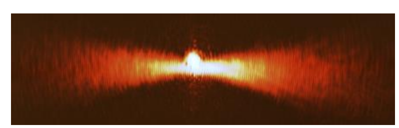


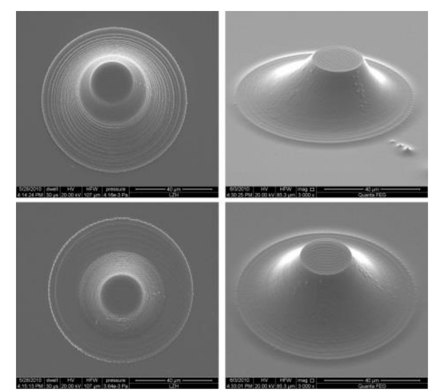


$$\varepsilon(r) = \begin{cases} \varepsilon_0, & r > R_{sh} \\ \varepsilon_0 \left(\dfrac{R_{sh}}{r} \right)^2, & R_c \le r \le R_{sh} \\ \varepsilon_c + \imath\gamma, & r < R_c, \end{cases}$$

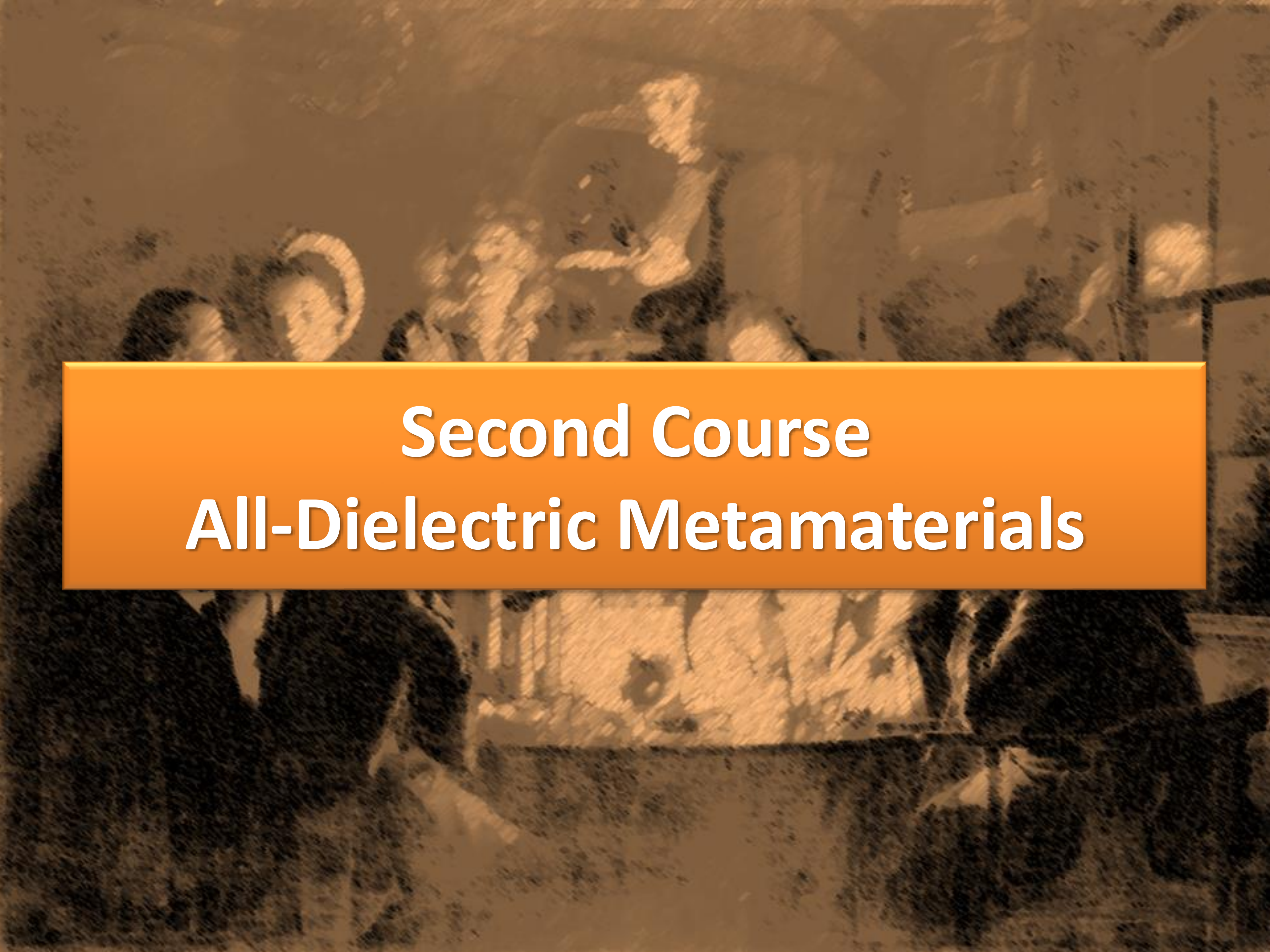


leakage radiation microscopy



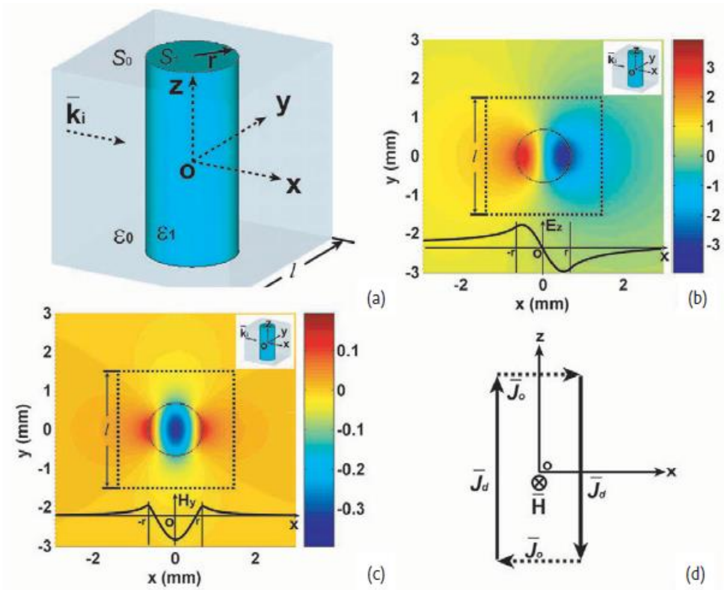


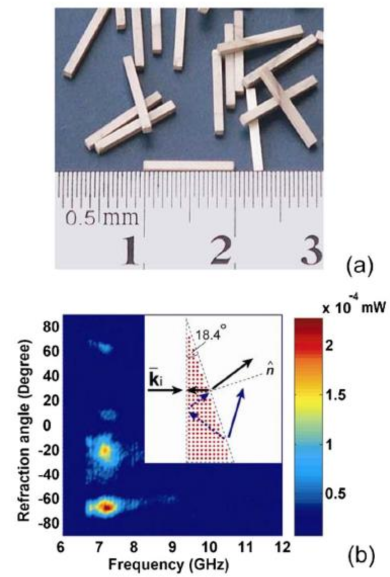
Argyropoulos et al., JOSAB (2010); Cheng et al., New J. Phys. (2010); Photo by Laser Zentrum Hannover



Second Course
All-Dielectric Metamaterials

Cylindrical Dielectric Resonators

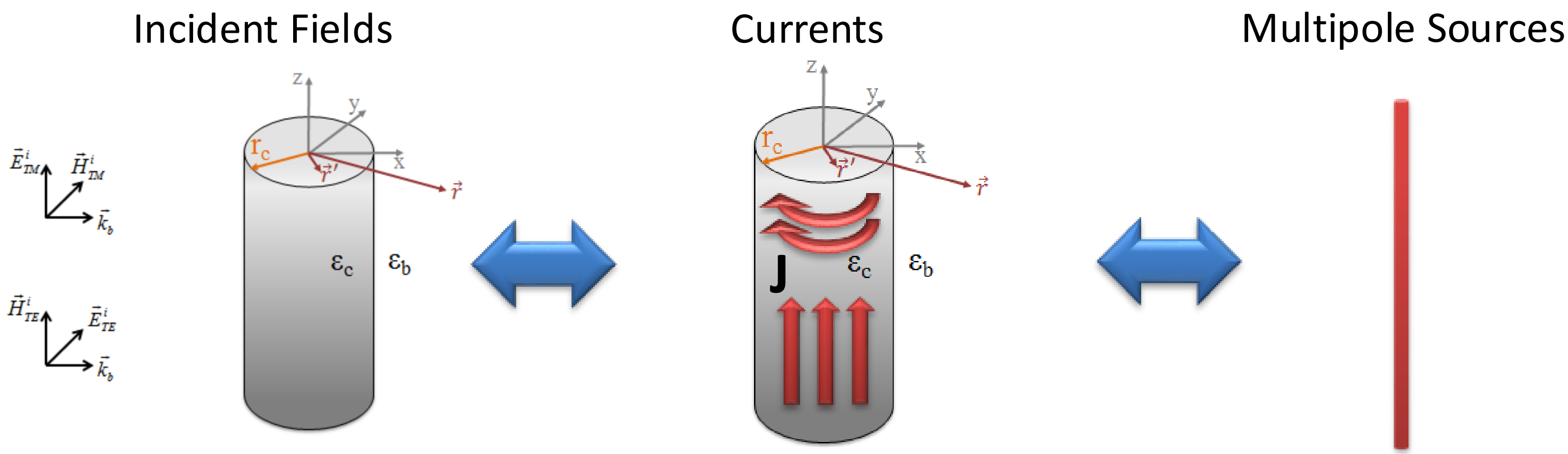


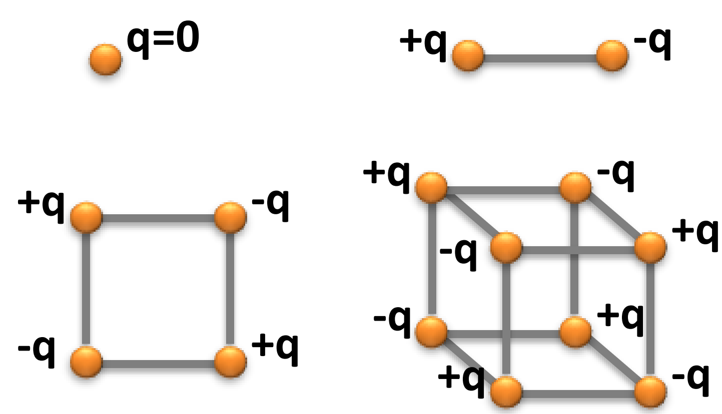


- Avoid plasmonic losses
- Physical principle: polarization currents
- Need high-ε materials (e.g. Si at optical frequencies)
- Anisotropic response
- → Retrieve effective medium parameters

Peng et al., PRL (2007)

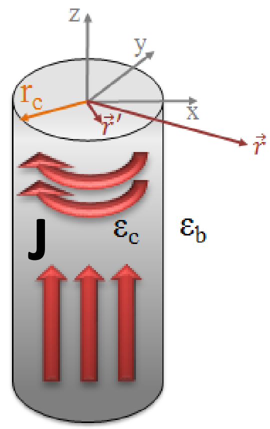
Principle of Multipole Expansion

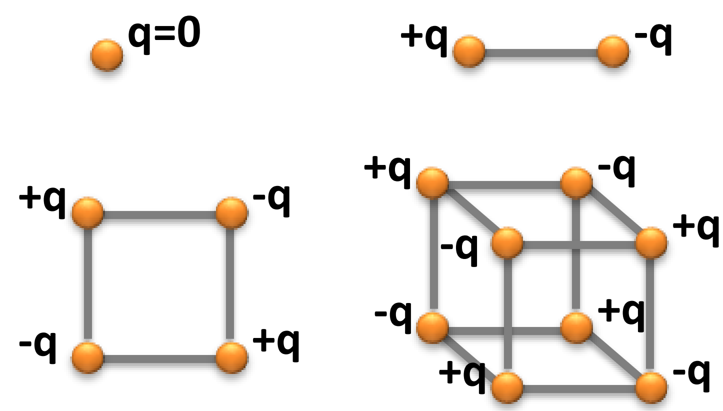




- Manipulate equivalent point-like elements
- Compatible with existing models (Maxwell-Garnett, etc.)

Multipole Expansion





$$\vec{A}(\vec{r}) = \mu_0 \int G(\vec{r}-\vec{r}')\vec{J}(\vec{r}')dS',$$

$$\varphi(\vec{r}) = \frac{1}{\varepsilon_0 \varepsilon_b}\left[\int G(\vec{r}-\vec{r}')\rho(\vec{r}')dS' + \oint G(\vec{r}-\vec{r}')\sigma(\vec{r}')dl'\right]$$

$$multipoles \sim J \cdot r_c^n$$

$$\vec{p} = \frac{j}{\omega}\int_S \vec{J}(\vec{r}')dS'$$

$$\vec{m} = \frac{1}{2}\int_S \vec{r}' \times \vec{J}(\vec{r}')dS'$$

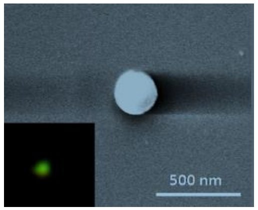
$$\overleftrightarrow{Q} = \frac{j}{\omega}\int_S \left[\vec{J} \otimes \vec{r}' + \vec{r}' \otimes \vec{J}\right]dS'$$

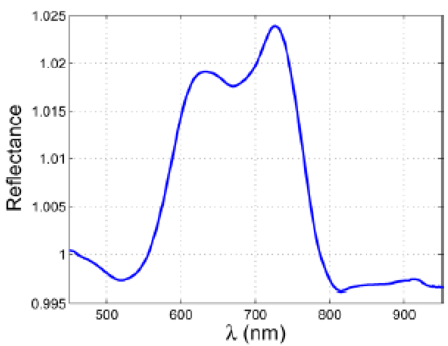
$$\vec{p} = \varepsilon_0 \varepsilon_b \overleftrightarrow{\alpha}^e \cdot \vec{E}^i$$

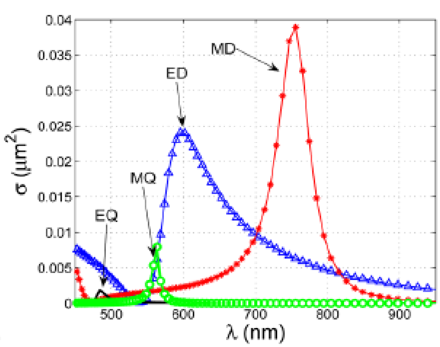
$$\vec{m} = \overleftrightarrow{\alpha}^m \cdot \vec{H}^i$$

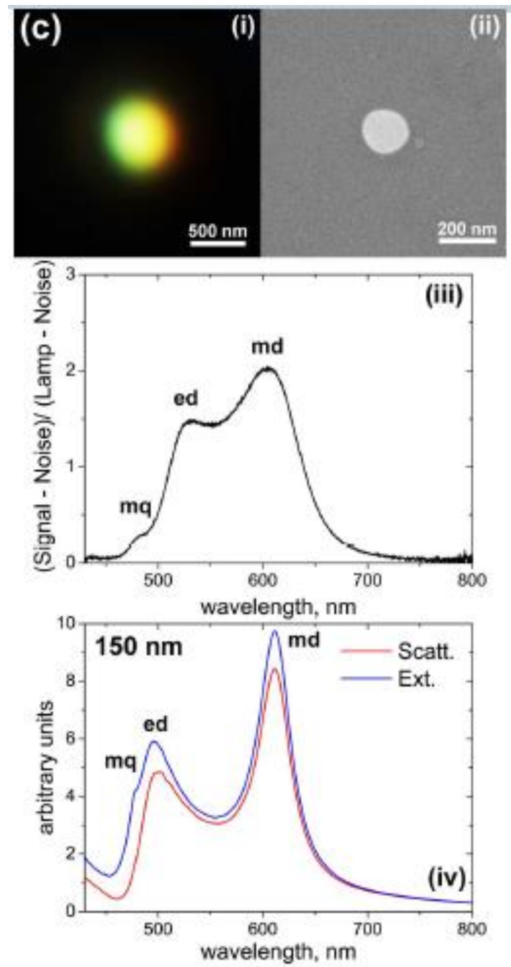
$$\overleftrightarrow{Q} = \varepsilon_0 \varepsilon_b \overleftrightarrow{\alpha}^q \cdot \vec{E}^i$$

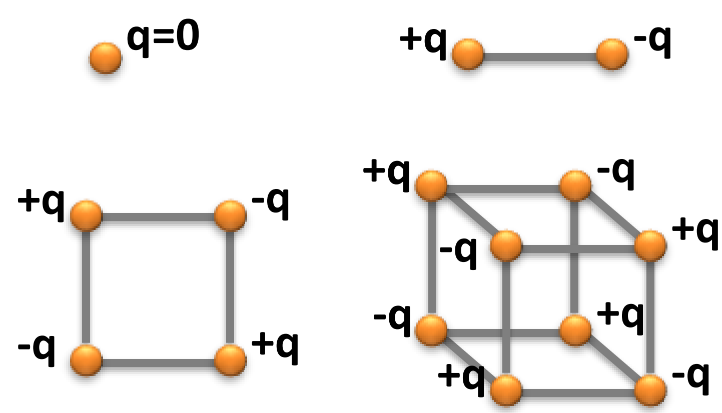
Verified for Si Nanoparticles











Evlyukhin et al., Nanoletters (2012), Kuznetsov et al., Nature Scientific Reports (2012)

$$\vec{f}_0 + \vec{f}_1 = \frac{ik^2}{4\varepsilon_0} \cdot \frac{1}{c_b} \vec{m} \times \hat{n} + \frac{ik^2}{4\varepsilon_0 c_b} \vec{p} c_b$$

$$= \frac{ik^2}{4\varepsilon c_b} \left(\vec{p} c_b + \vec{m} \times \hat{n} \right) =$$

$$\frac{1}{m^2} \cdot \frac{m}{F} \cdot \frac{V}{m} \cdot c_b \cdot \frac{m}{s}$$

$$\vec{E} = \frac{ik^2}{4\varepsilon_0 c_b} \left[\vec{p} c_b - \vec{m} \times \hat{n} \right] \underbrace{\sqrt{\frac{2}{\pi}}\, e^{-i\pi/4} \frac{e^{+ikr}}{\sqrt{kr}}}_{h}$$

$$Z_b = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{\mu_b}{\varepsilon_b}} = Z_0 / \sqrt{\varepsilon_b}$$

$$\frac{1}{\varepsilon_0 c_b} = \frac{1}{\varepsilon_0 \cdot \frac{c}{\sqrt{\varepsilon_b}}} = \frac{\sqrt{\varepsilon_b}}{\varepsilon_0 \cdot \frac{1}{\sqrt{\varepsilon_0 \mu_0}}} = \frac{\sqrt{\varepsilon_b}}{\sqrt{\frac{\varepsilon_0}{\mu_0}}} = Z_0 \sqrt{\varepsilon_b}$$

$$1/Z_0$$

$$Z_b \cdot \sqrt{\varepsilon_b} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\varepsilon_b} = \frac{\sqrt{\varepsilon_b}\sqrt{\varepsilon_0 \mu_0}}{\sqrt{\varepsilon_0}}$$

$$\vec{E} = \sqrt{\varepsilon_b} \cdot Z_0 \cdot \frac{ik^2}{4} \left[\vec{p} c_b - \hat{n} \times \vec{m} \right] \sqrt{\frac{2}{\pi}}\, e^{-i\pi/4} \frac{e^{+ikr}}{\sqrt{kr}}$$

$$\vec{H} = -\frac{1}{Z_b} \vec{E} \times \hat{n} = -\frac{\sqrt{\varepsilon_b}}{Z_0} \cdot \sqrt{\varepsilon_b} \cdot Z_0 \cdot \frac{ik^2}{4} \left[\vec{p} c \times \hat{n} + (\vec{m} \times \hat{n}) \times \hat{n} \right]$$

$$= \varepsilon_b \frac{ik^2}{4} \left[(\hat{n} \times \vec{p} c_b) + (\hat{n} \times \vec{m} \times \hat{n}) \right] \sqrt{\frac{2}{\pi}}\, e^{-i\pi/4} \frac{e^{+ikr}}{\sqrt{kr}}$$

$$\vec{p} \begin{cases} \vec{E} = Z_0 \frac{ik^2}{4} \vec{p} c \cdot h \\ \vec{H} = \frac{ik^2}{4} \cdot \hat{n} \times \vec{p} c \cdot h \end{cases}$$

$$\vec{m} \begin{cases} \vec{H} = \frac{ik^2}{4} \vec{m} \cdot h \\ \vec{E} = -Z_0 \frac{ik^2}{4} \cdot \hat{n} \times \vec{m} \cdot h \end{cases}$$

$$\vec{E} = \sqrt{\varepsilon_b} Z_0 \cdot \frac{ik^2}{4} \left(\vec{p} c_b - \hat{n} \times \vec{m} \right) \sqrt{\frac{2}{\pi}}\, e^{-i\pi/4} \frac{e^{+ikr}}{\sqrt{kr}}$$

$$\vec{H} = \varepsilon_b \frac{ik^2}{4} \left(\hat{n} \times \vec{p} c_b + \vec{m} \right) \sqrt{\frac{2}{\pi}}\, e^{-i\pi/4} \frac{e^{+ikr}}{\sqrt{kr}}$$

Let $\vec{E} = h \cdot (b_0 + 2 b_1 \cos\varphi) \hat{z}$

$$\hat{z} b_0 = \sqrt{\varepsilon_b} Z_0 \cdot \frac{ik^2}{4} \cdot c_b \cdot \vec{p} \rightarrow$$

$$= \frac{ik^2}{4\varepsilon_0} \vec{p} \rightarrow$$

$$\rightarrow \frac{\vec{p}}{\varepsilon_0} = +\hat{z} \frac{4}{ik^2} b_0$$

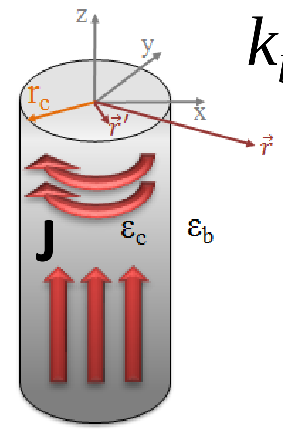
$$(\hat{x}\cos\varphi + \hat{y}\sin\varphi) \times (m_x \hat{x} + m_y \hat{y}) = \hat{z}\cos\varphi\, m_y - \hat{z} m_x \sin\varphi$$

$$2 b_1 \cos\varphi \hat{z} = -\sqrt{\varepsilon_b} Z_0 \frac{ik^2}{4} \hat{x} \times \vec{m}$$

$$= -\hat{z} \sqrt{\varepsilon_b} \frac{Z_0}{\sqrt{\varepsilon_b}} \frac{ik^2}{4} m_y \cos\varphi \qquad = -\hat{z}\, \varepsilon_b \cdot Z_b$$

$$Z_b \vec{m} = -\hat{y} \frac{4 b_1}{ik^2 \varepsilon_b}$$

Multipole Expressions



$$k_b r' = 1$$

$$\vec{E}_{TM} / Z_b = +k_b^2 \hat{z}\left[p_z c_b G + 2 m_\varphi jG' \right]$$

$$\vec{H}_{TM} = +2k_b^2 \left[\hat{n} m_n \left(G + G'' \right) + \hat{\varphi} m_\varphi \left(G + \frac{G'}{k_b r} \right) \right] + k_b^2 \hat{\varphi}\left[p_z c_b jG' \right]$$

$$\vec{E}_{TE} / Z_b = +k_b^2 \left[\hat{n} p_n c_b \left(G + G'' \right) + \hat{\varphi} p_\varphi c_b \left(G + \frac{G'}{k_b r} \right) \right] - k_b^2 \hat{\varphi}\left[m_z jG' \right]$$

$$- \frac{k_b^2}{2} \left[\hat{n} \omega Q_{nn} \left(G' + G''' \right) + \hat{n} \omega Q_{\varphi\varphi} \left(\frac{G''}{k_b r} - \frac{G'}{(k_b r)^2} \right) + \hat{\varphi} \omega Q_{n\varphi} \left(G' - \frac{2G'}{(k_b r)^2} + \frac{2G''}{k_b r} \right) \right]$$

$$\vec{H}_{TE} = -k_b^2 \hat{z}\left[p_\varphi c_b jG' - m_z G - \frac{j}{2} \omega Q_{n\varphi} \left(G'' - \frac{G'}{k_b r} \right) \right]$$

$$\vec{E}_{3D} / Z = +k^3 \left[\left(\hat{n} \cdot \vec{p}c \right) \hat{n} G'' + \left(\vec{p}c \right) G + \left(\hat{n} \times \vec{m} \right) jG' \right]$$

$$\vec{H}_{3D} = -k^3 \left[\left(\hat{n} \cdot \vec{m} \right) \hat{n} G'' + \vec{m} G + \left(\hat{n} \times \vec{p}c \right) jG' \right]$$

Kallos et al. (submitted)

Fields – Multipoles Correlation

$$Multipoles \sim J \cdot r_c^n$$

$$\vec{E}/Z_b \overset{k_b r \ll 1}{\approx} +k_b^2\left[+\hat{\varphi}\left(p_\varphi c_b + m_z - \frac{j}{2}\omega Q_{n\varphi}\right) + \hat{z}\left(p_z c_b - 2m_\varphi\right)\right]G_\infty$$

$$\vec{H} \overset{k_b r \ll 1}{\approx} +k_b^2\left[+\hat{z}\left(p_\varphi c_b + m_z - \frac{j}{2}\omega Q_{n\varphi}\right) - \hat{\varphi}\left(p_z c_b - 2m_\varphi\right)\right]G_\infty$$

$$Mie \sim \left(r_c/\lambda_0\right)^n$$

$$\vec{E}_{TM} \overset{k_b r \ll 1}{\approx} -\hat{z}\left[b_0 + 2b_1\cos\varphi + 2b_2\cos 2\varphi\right]4jG_\infty$$

$$\vec{H}_{TE} \overset{k_b r \ll 1}{\approx} -\hat{z}\left[a_0 + 2a_1\cos\varphi + 2a_2\cos 2\varphi\right]4jG_\infty$$

$$\vec{p}_{TM} = +\hat{z}\frac{4b_0}{jk_b^2 Z_b c_b} \qquad \vec{m}_{TM} = -\hat{y}\frac{4b_1}{jk_b^2 Z_b} \qquad \overline{\overline{Q}}_{TM} = \overline{\overline{0}}$$

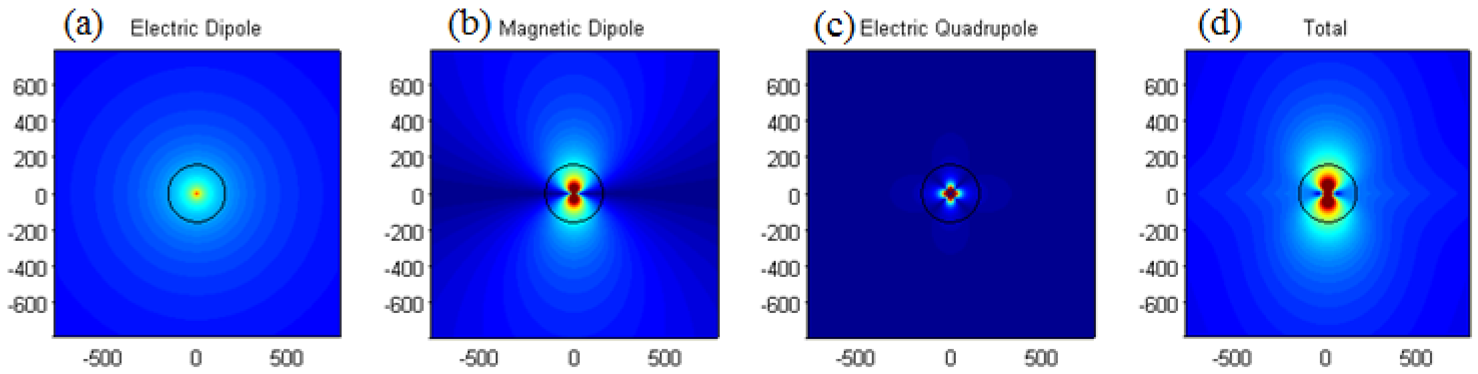
$$\vec{p}_{TE} = +\hat{y}\frac{8a_1}{jk_b^2 c_b} \qquad \vec{m}_{TE} = +\hat{z}\frac{4a_0}{jk_b^2} \qquad \overline{\overline{Q}}_{TE} = \left(\hat{x}\hat{y} + \hat{y}\hat{x}\right)\frac{16a_2}{k_b^3 c_b}$$

$$\vec{p} = \varepsilon_0\varepsilon_b \overline{\overline{\alpha}}^e \cdot \vec{E}^i$$

$$\vec{m} = \overline{\overline{\alpha}}^m \cdot \vec{H}^i$$

$$\overline{\overline{Q}} = \varepsilon_0\varepsilon_b \overline{\overline{\overline{\alpha}}}^q \cdot \vec{E}^i$$

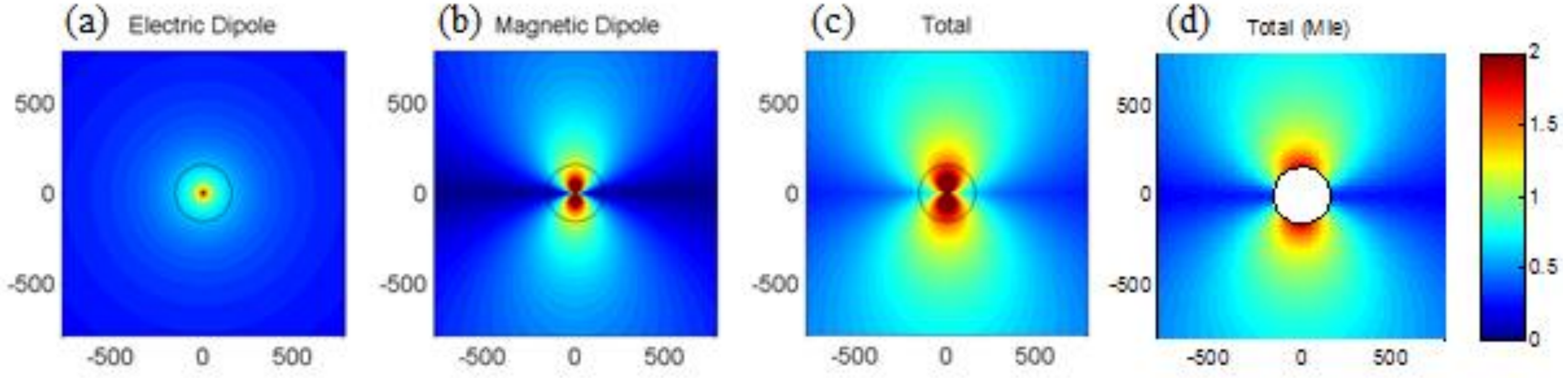
Multipole Fields - TE



- r_c =158 nm cylinder
- Lossless Silicon (ε=18)
- λ=10*r_c

Kallos et al. (submitted)

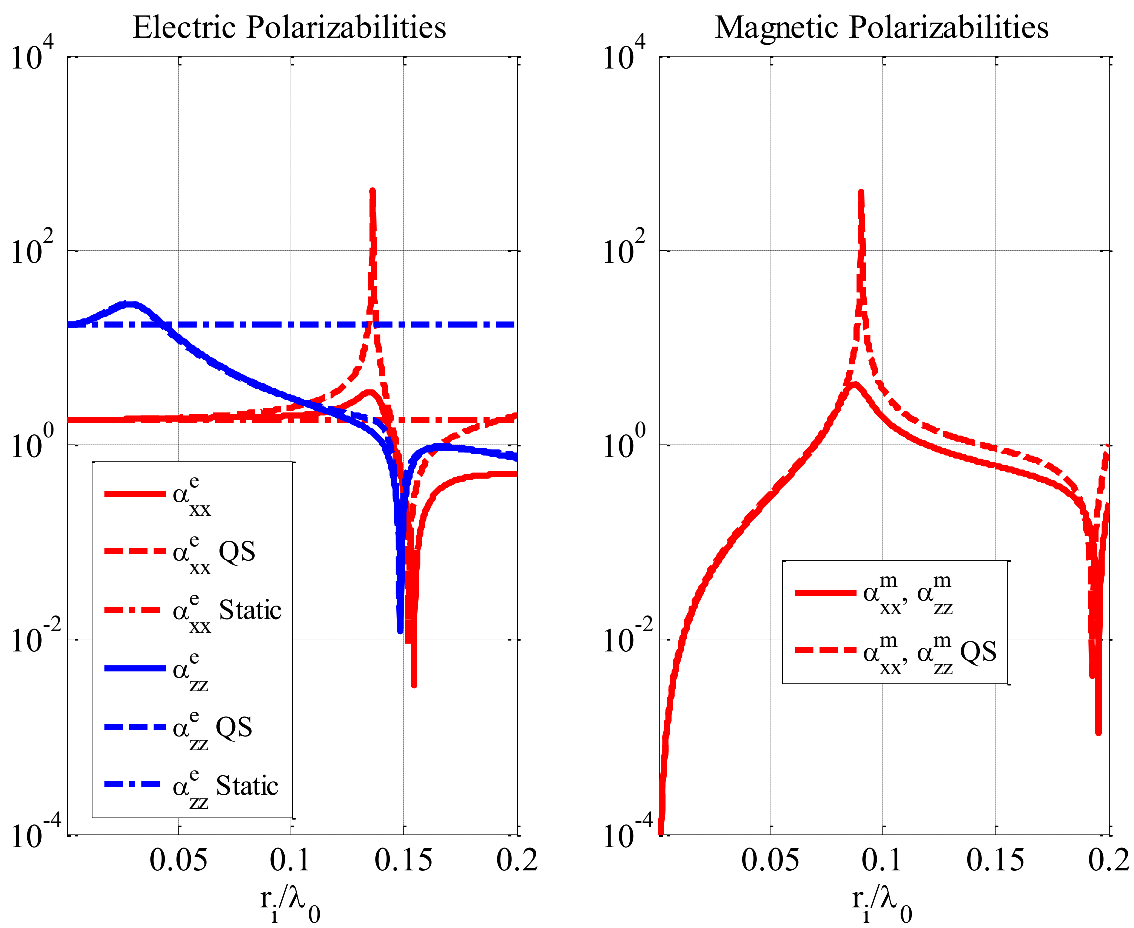
Multipole Fields - TM



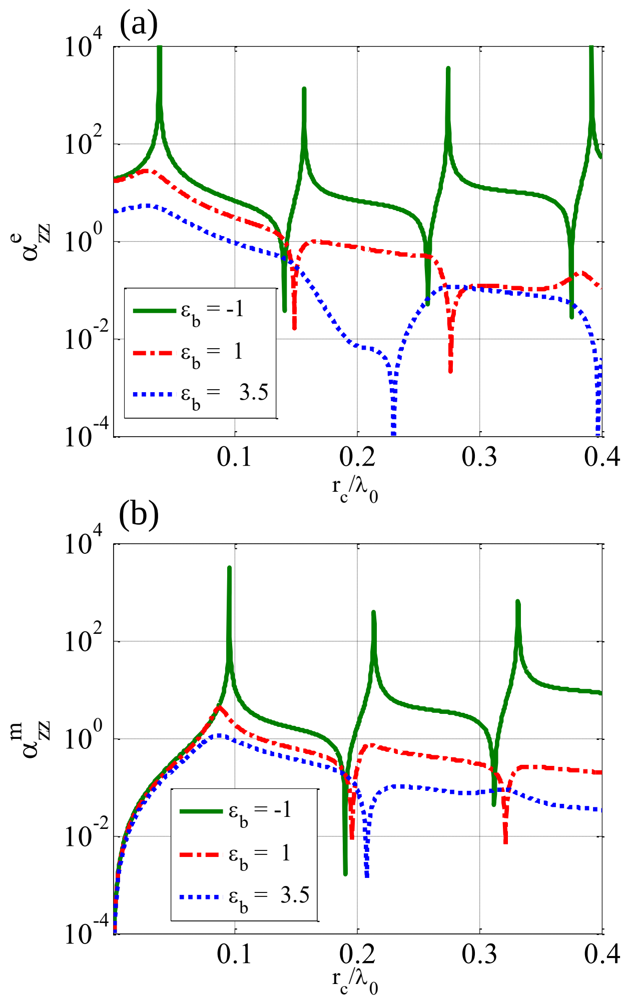
- r_c =158 nm cylinder
- Lossless Silicon (ε=18)
- $\lambda=4*r_c$

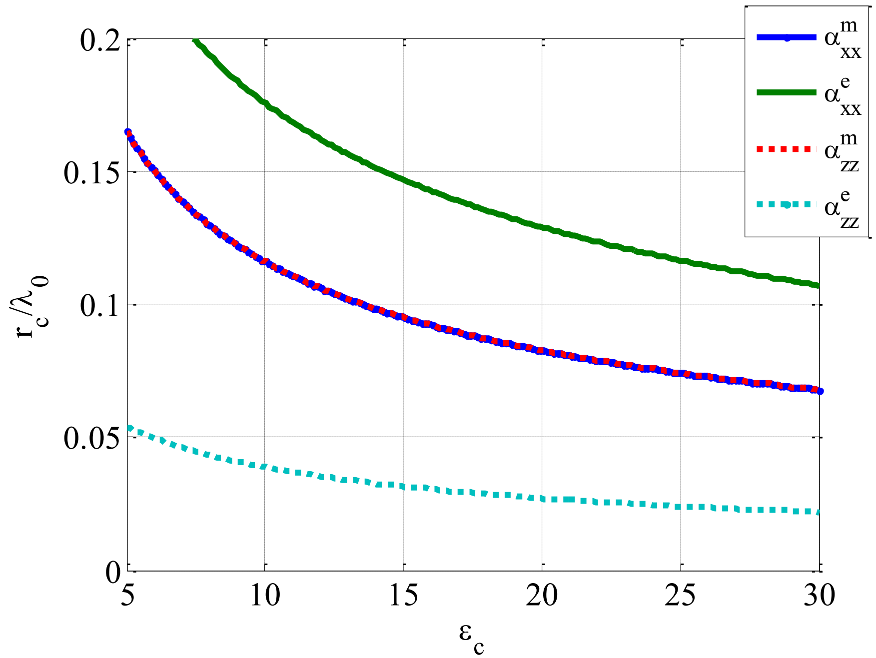
Kallos et al. (submitted)

Resonance Hunter



Resonance Hunter

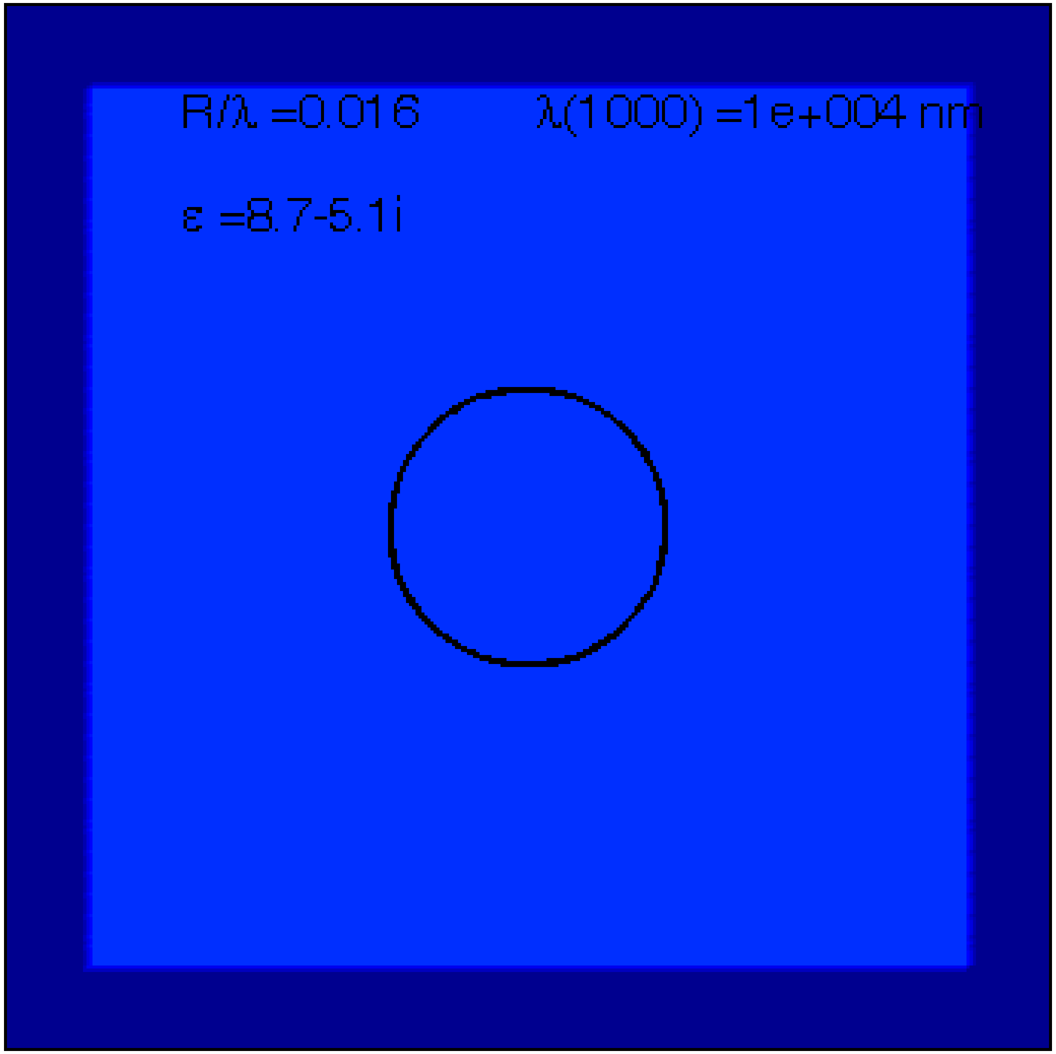




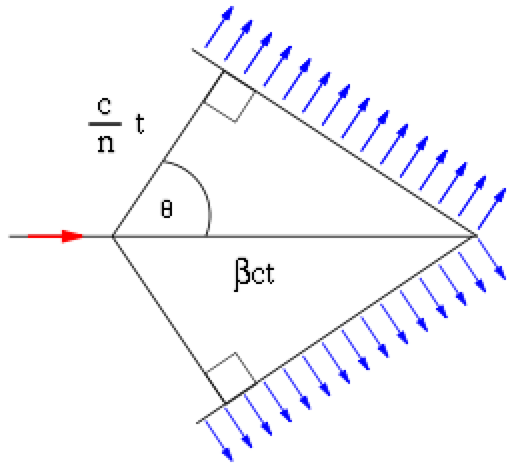
Kallos et al. (under review)

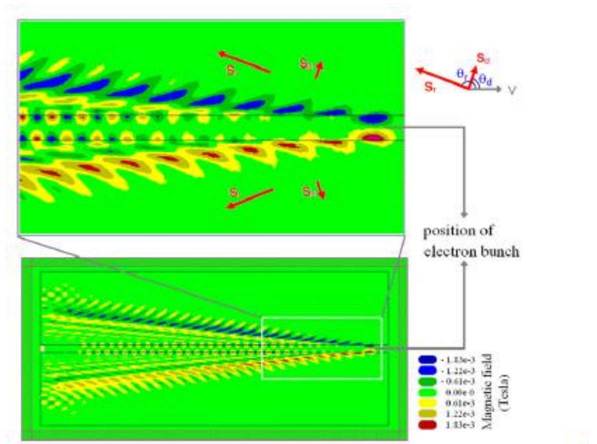
COMSOL Simulations

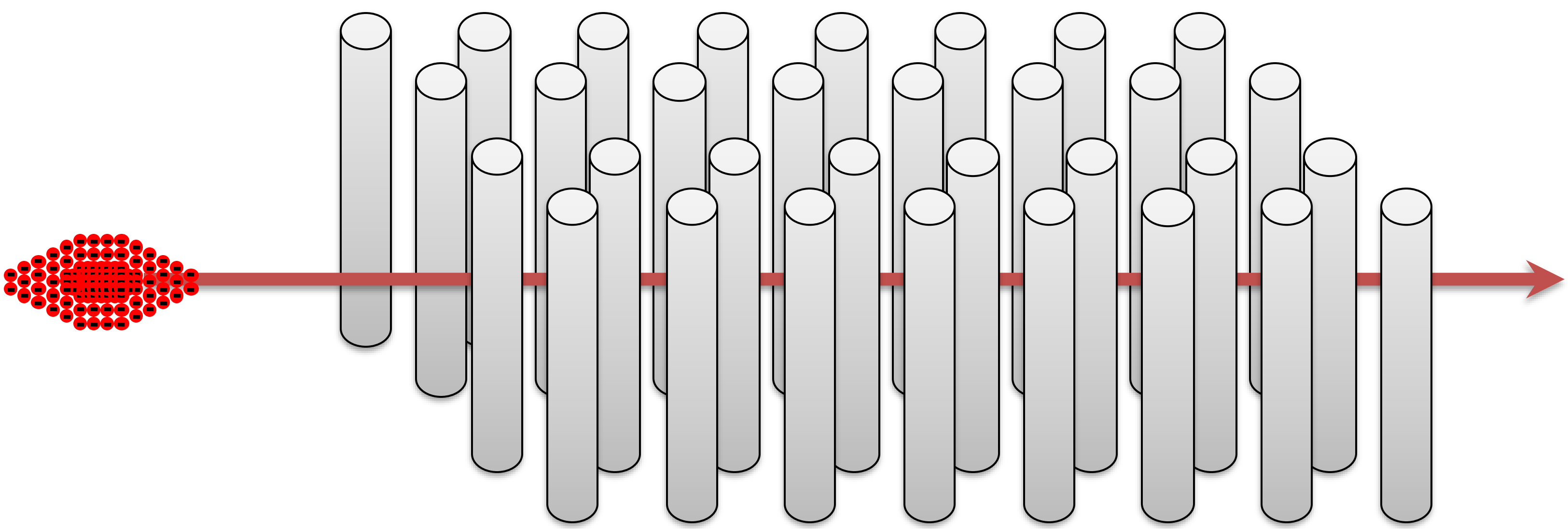
Silicon Nanorod



Inverse Cherenkov Radiation

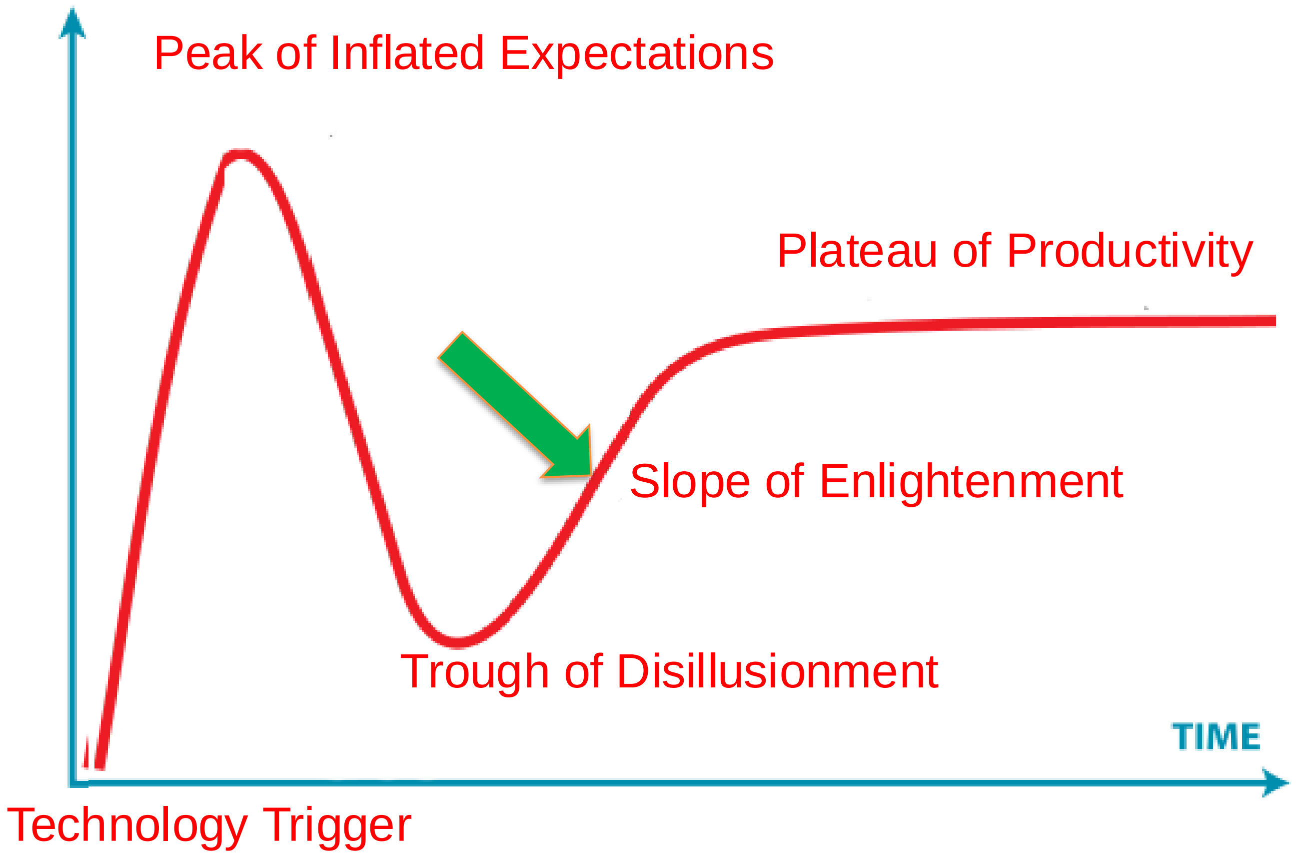






Third Course
Commercial Metamaterials

The Hype Curve



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The Problem

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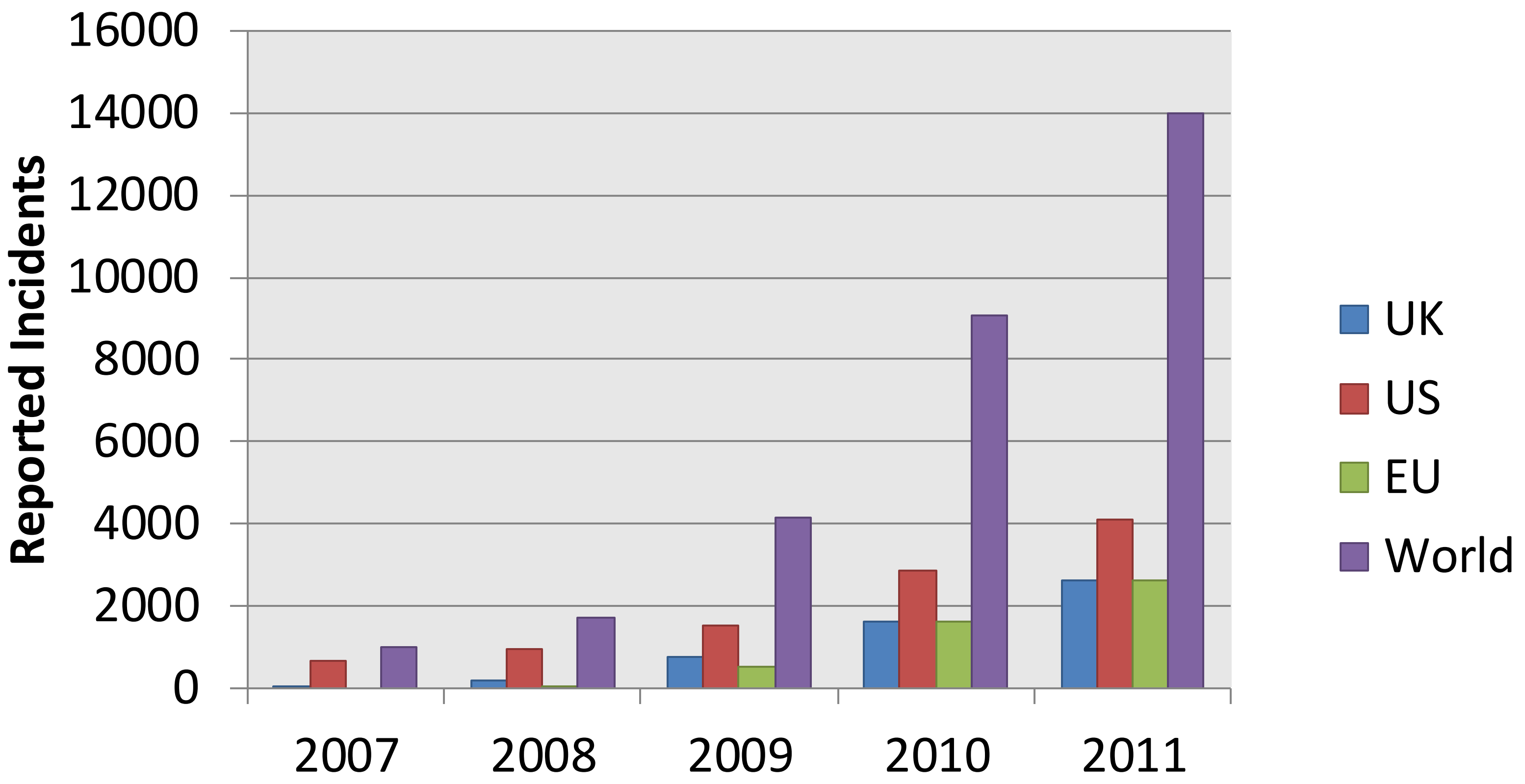
The Problem



Source: Laser Interference Seminar, Eurocontrol, Brussels, Belgium (2011)

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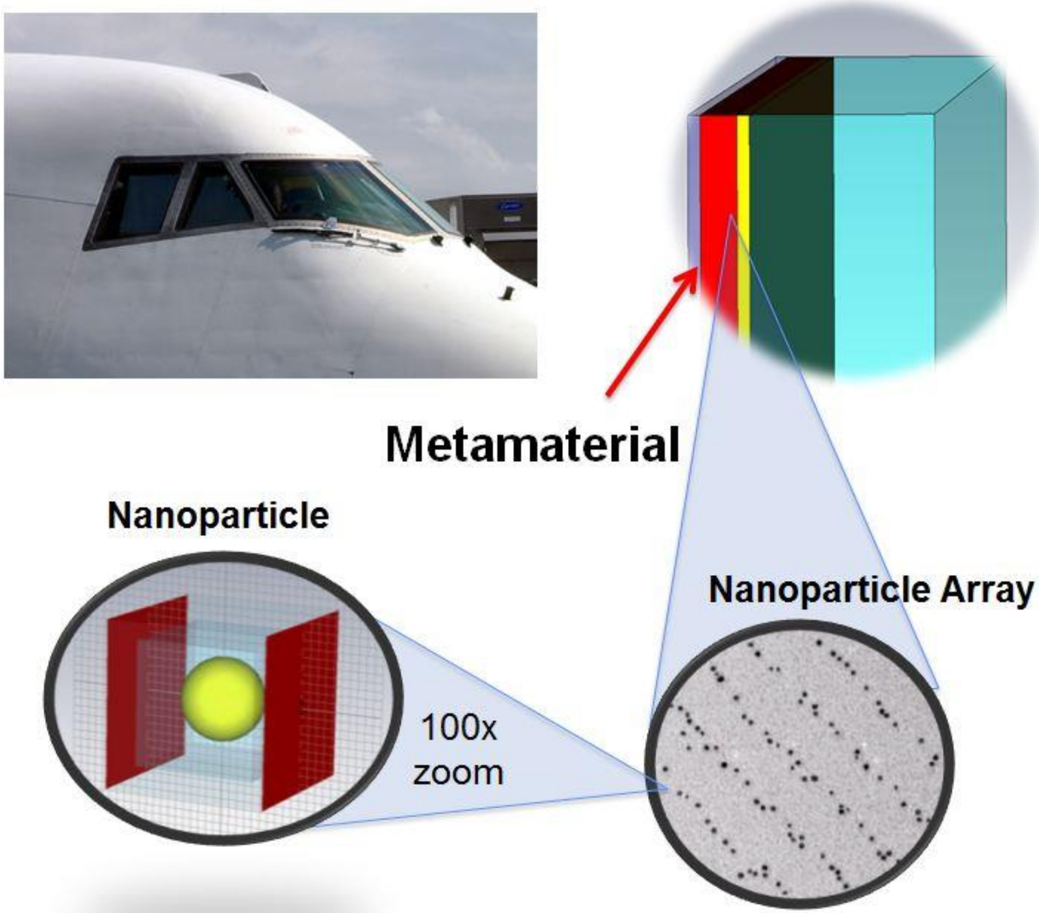
The Solution



- **OD≥2**
- **Transparent**
  (bandgap ~5-10nm)
- **Omnidirectional**
  (+/- 120^o)
- **Multiband**
  (green + blue)
- **Integrated transmission >70%**
- **Large Scale (~ m^2)**

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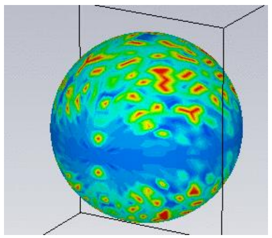
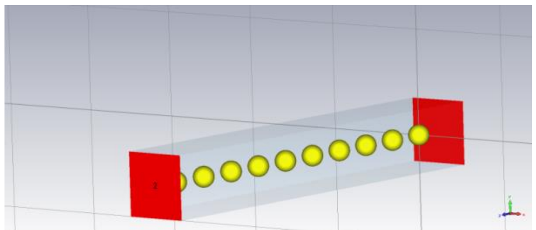
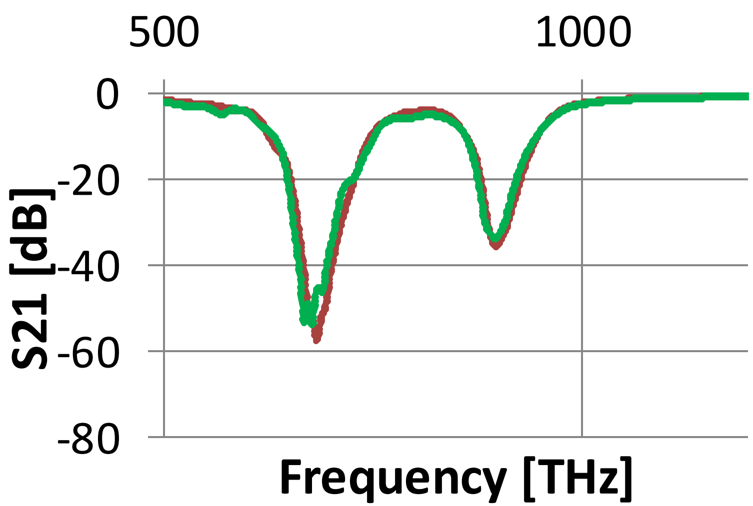
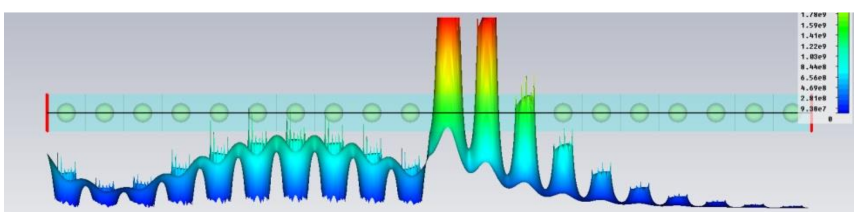
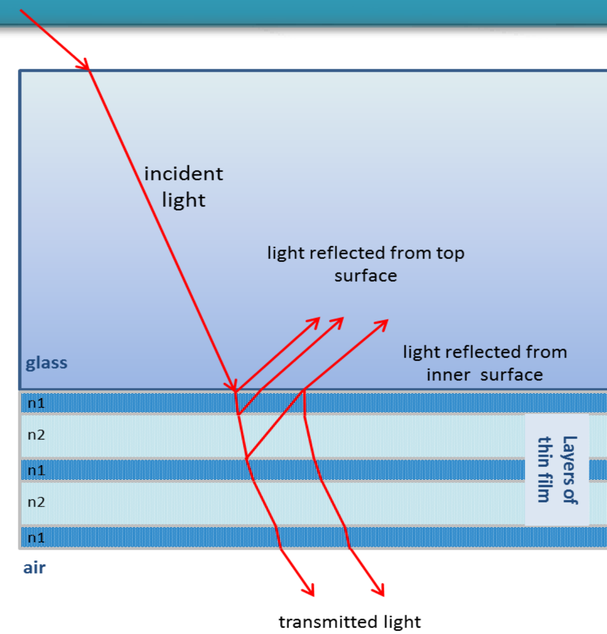
Filtering Ideas











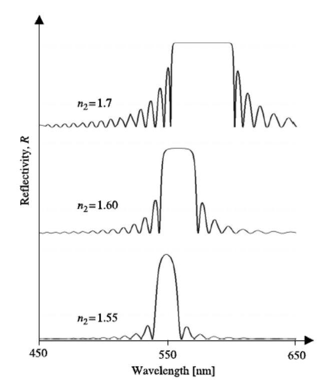


Figure 11.3 Reflectivity curves for different values of n_2. The reflectivity reaches 1 for each curve. Calculations are made for the parameters $n_1 = 1.5$, $p = 0.18\ \mu m$, and $L = 10\ \mu m$.

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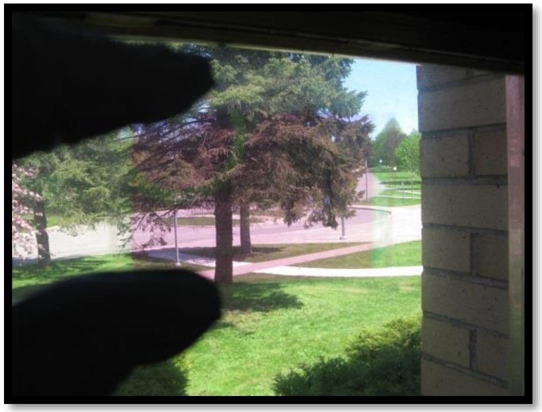
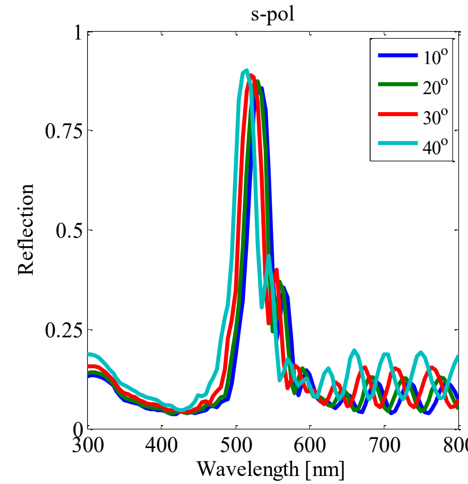
METAIR Film



Typical Goggles



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Partners: University of New Brunswick, University of Moncton

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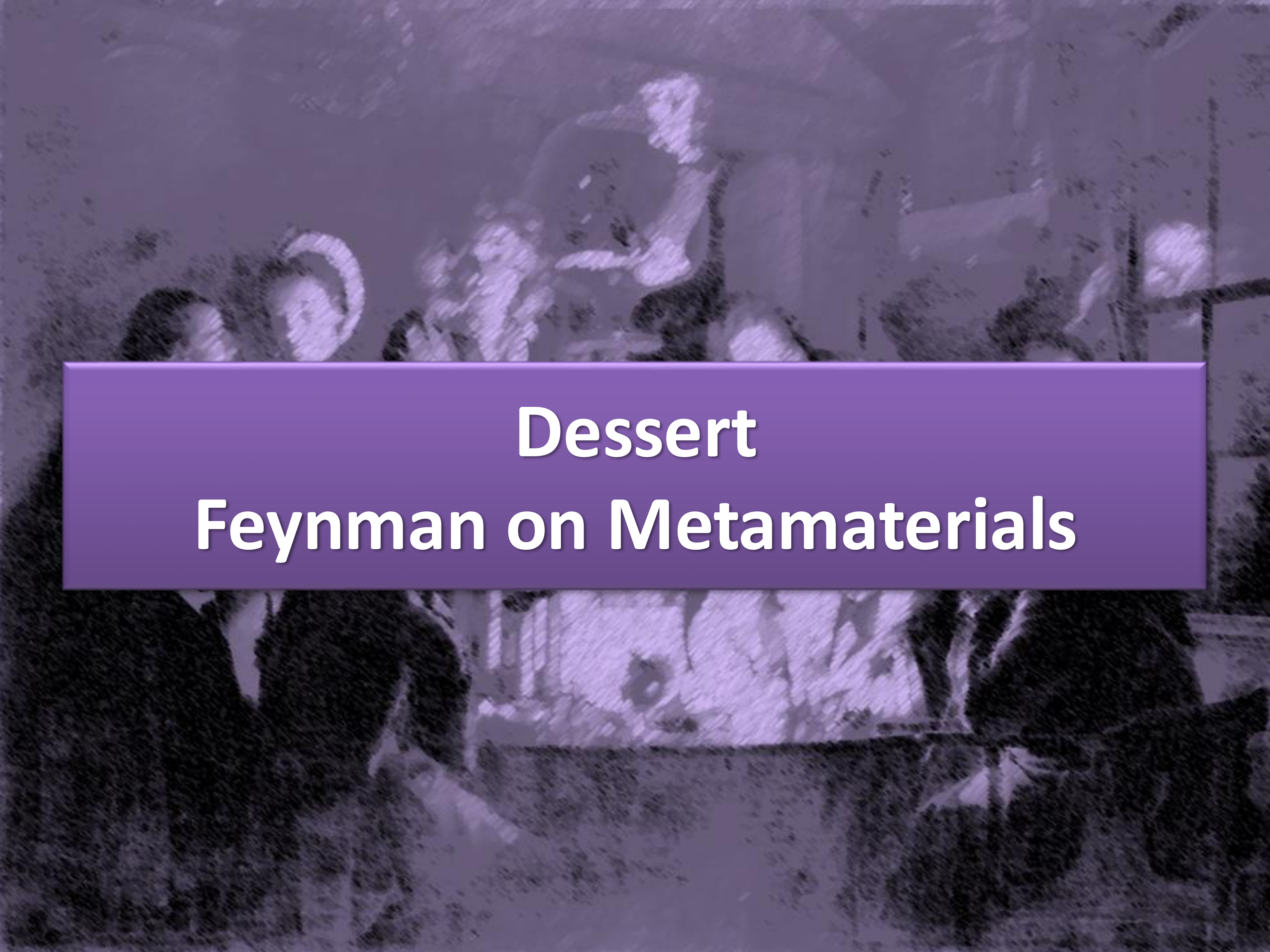
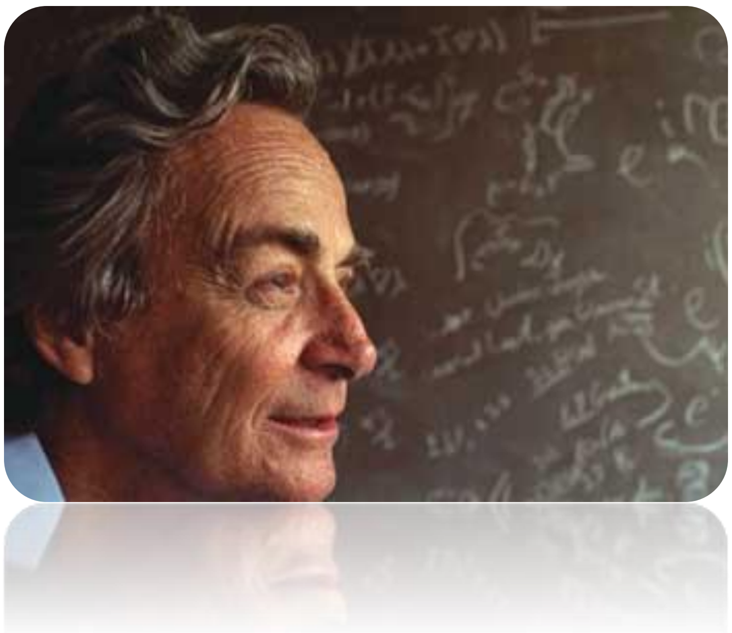
Dessert
Feynman on Metamaterials





"I can't see what exactly would happen,

but I can hardly doubt that when we have some control of the arrangement of things in the small scale,

we will get an enormously greater range of possible properties that substances can have."

1959

R. Feynman, There's Plenty of Room at the Bottom
http://www.zyvex.com/nanotech/feynman.html

Thank You!

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