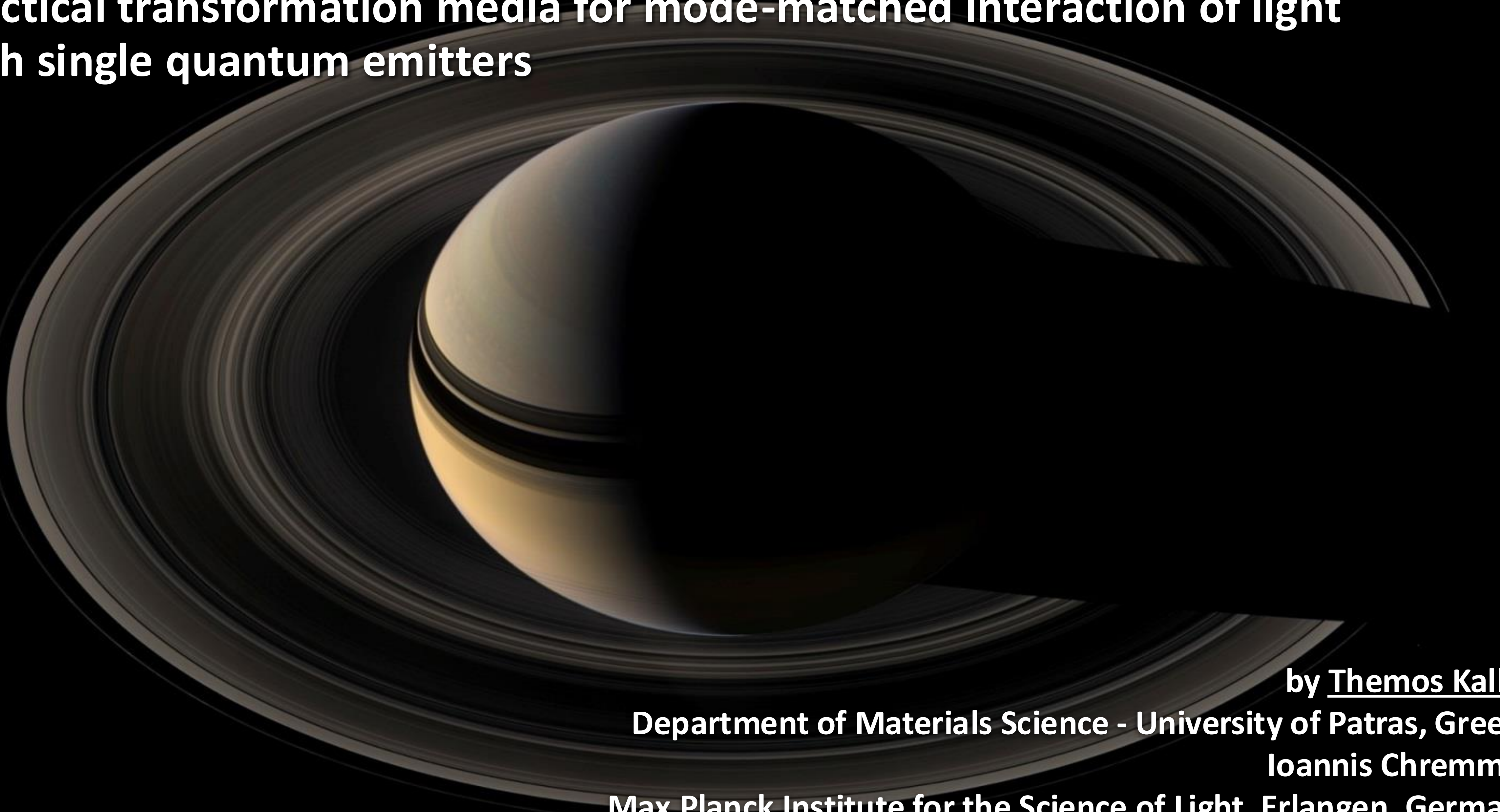
Practical transformation media for mode-matched interaction of light with single quantum emitters



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Practical transformation media for mode-matched interaction of light with single quantum emitters

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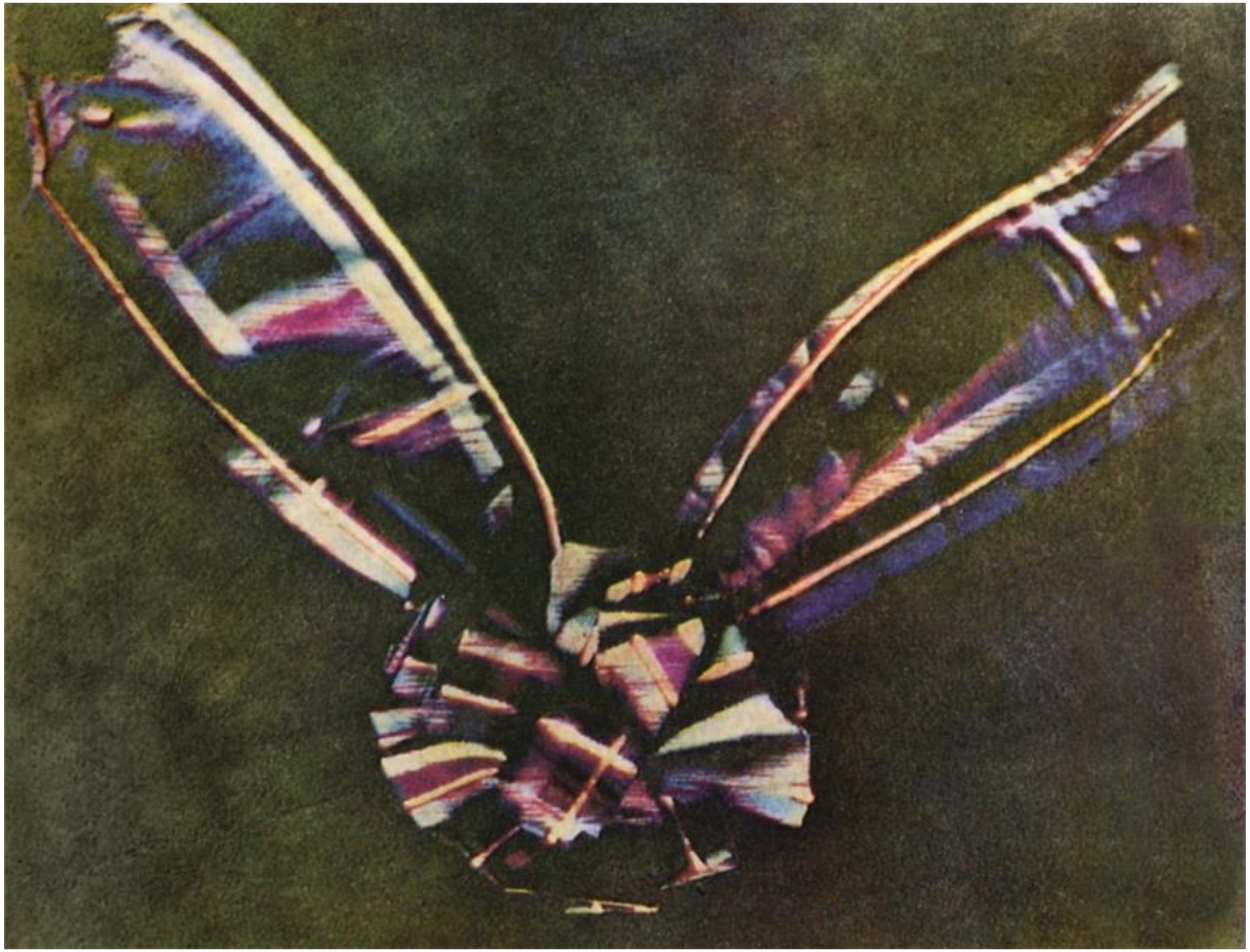
Outline

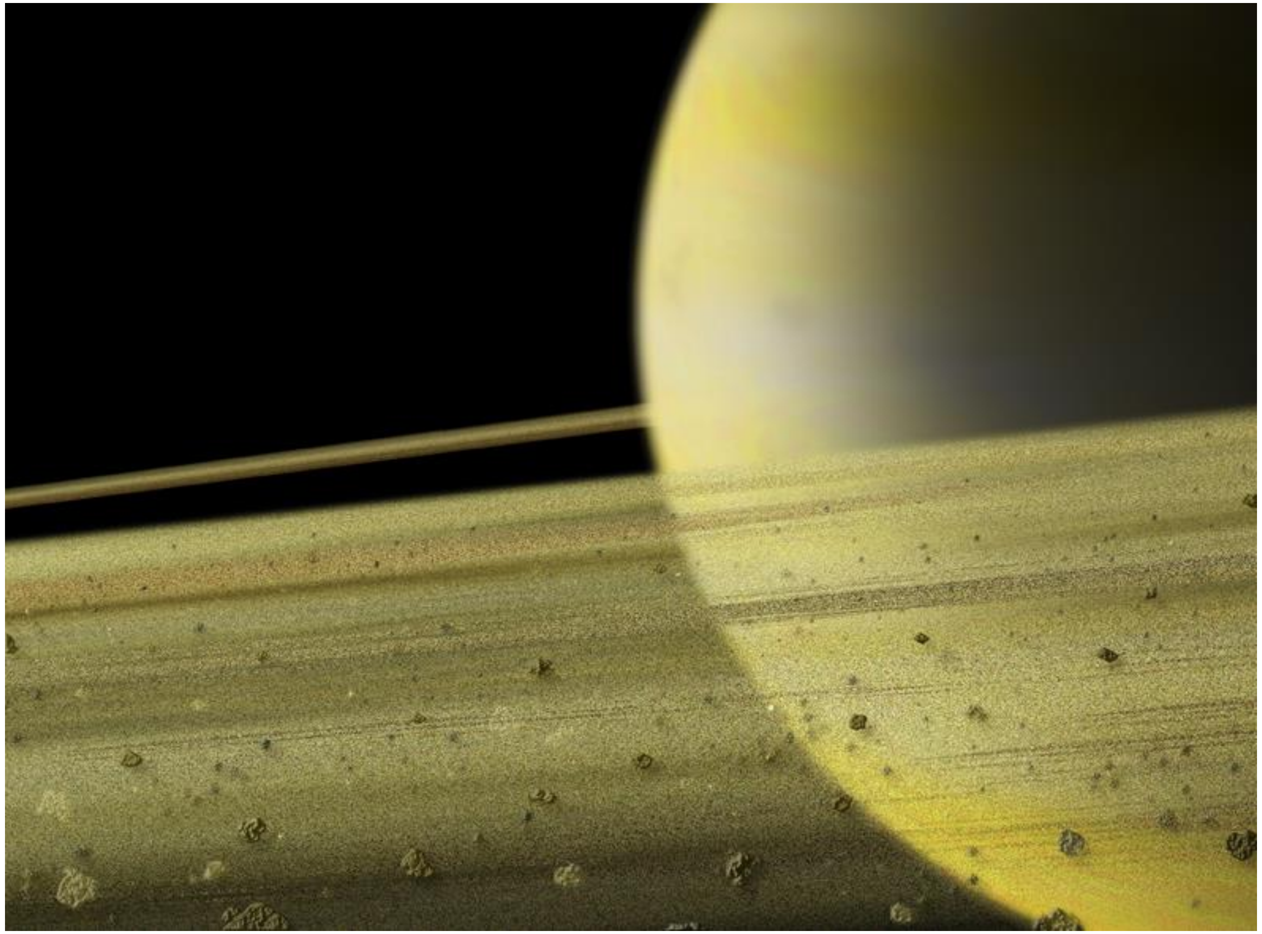
- Motivation
- Conformal maps for radial to parallel conversion of rays
- Practical index maps for 3D dipoles
- Dynamic analysis of arrays of dielectric resonators
- Awesome Feynman quote

3 things you (probably) didn't know about James Clerk Maxwell


JAMES
CLERK MAXWELL
1831-1879



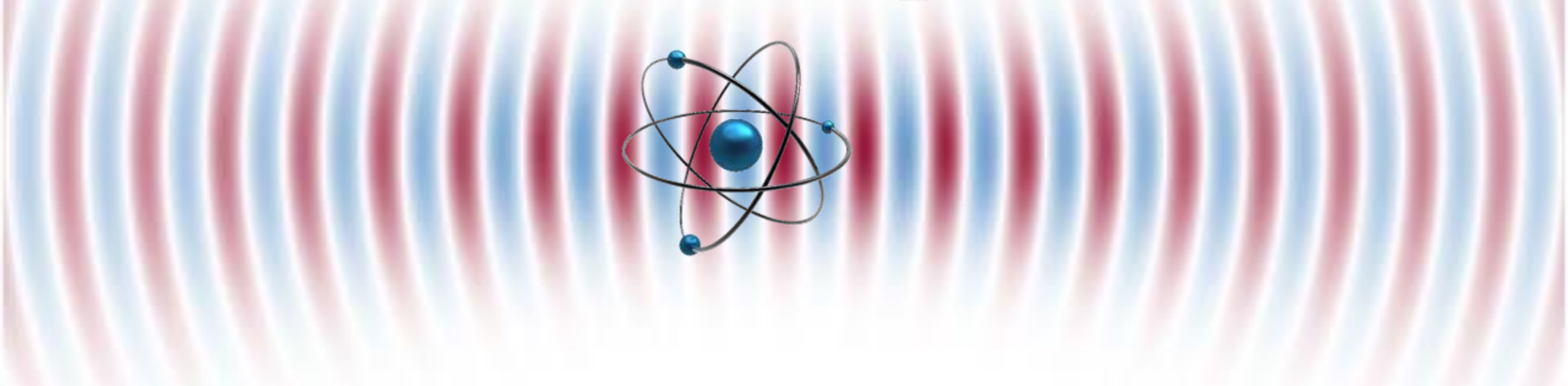
wikipedia



universetoday.com

Abstract

The efficient interaction of light with single quantum emitters (atoms, molecules and semiconductor nanocrystals) depends critically on the modal overlap between the incident and scattered photons. This usually calls for high-NA optics in order to match the dipole radiation pattern of the emitter in free space. Such a requirement can be alleviated if the emitter is embedded in a medium that shapes its radiation into a collimated output beam. We here present simple-to-realize, all-dielectric and isotropic transformation media that perform such a mode conversion.



Motivation and scope

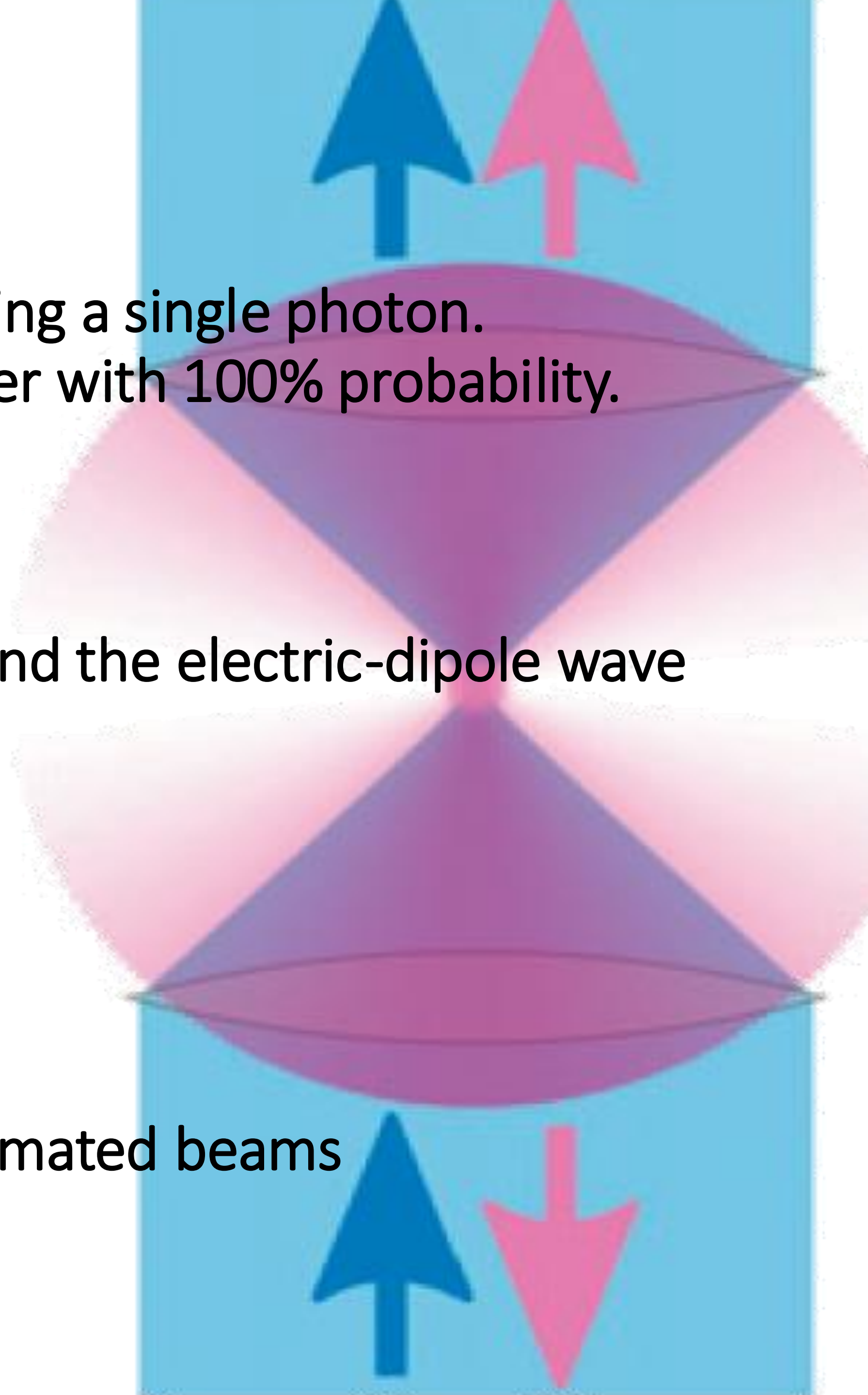
- Excite a single quantum emitter with 100% probability using a single photon.
- Detect single photons emitted by quantum emitter emitter with 100% probability.

These require….

- Increase the modal overlap between the incident beam and the electric-dipole wave associated with the emitter’s resonant transition.
- …still using standard limited-numerical-aperture optics.

We use transformation media to

- Achieve all the above in the bulk!
- Tailor the radiation pattern of quantum emitters into collimated beams
- Reach collection efficiencies > 99%

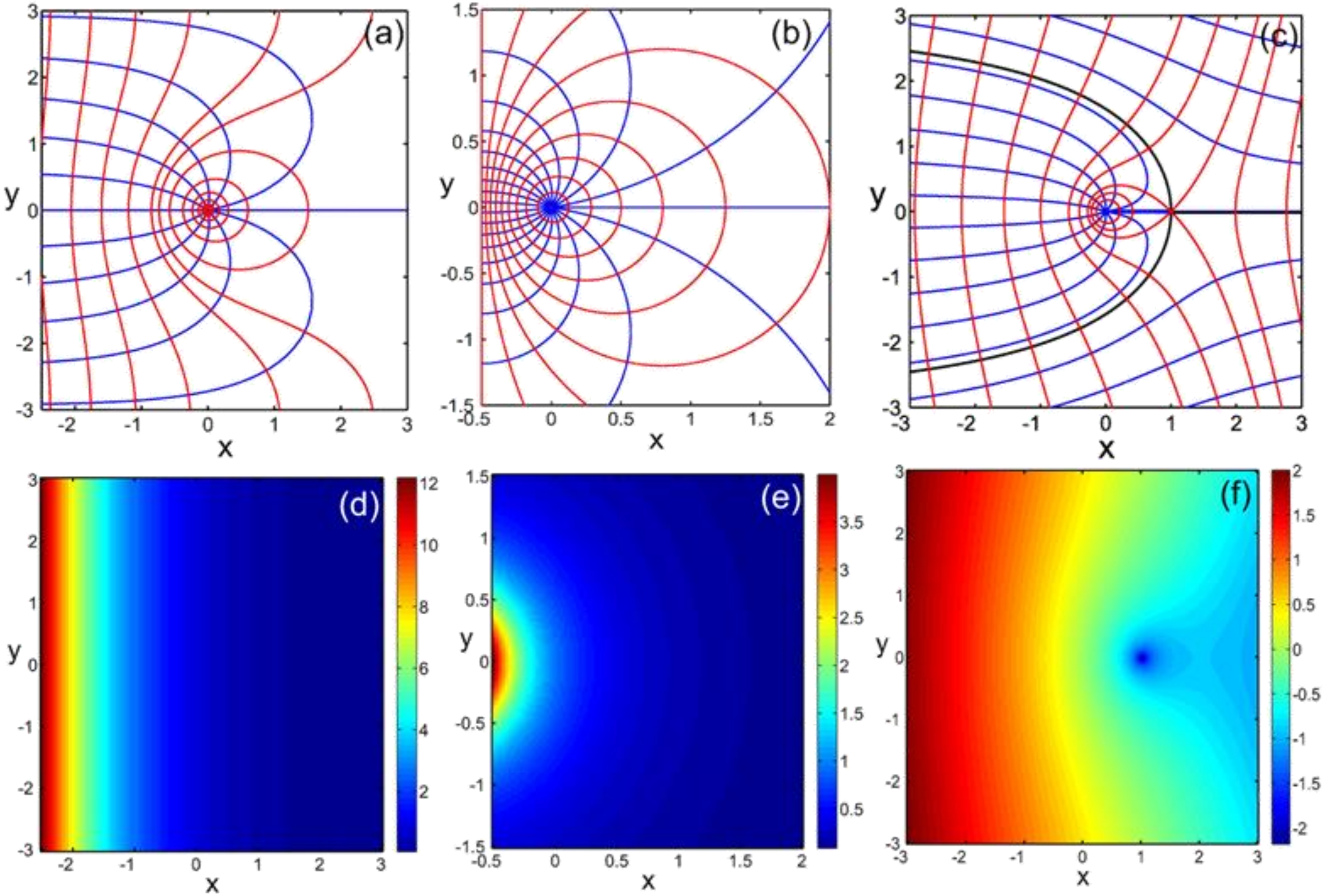


Conformal maps and the refractive index landscapes $|dW/dZ|$

$$W = e^{-Z} - 1$$

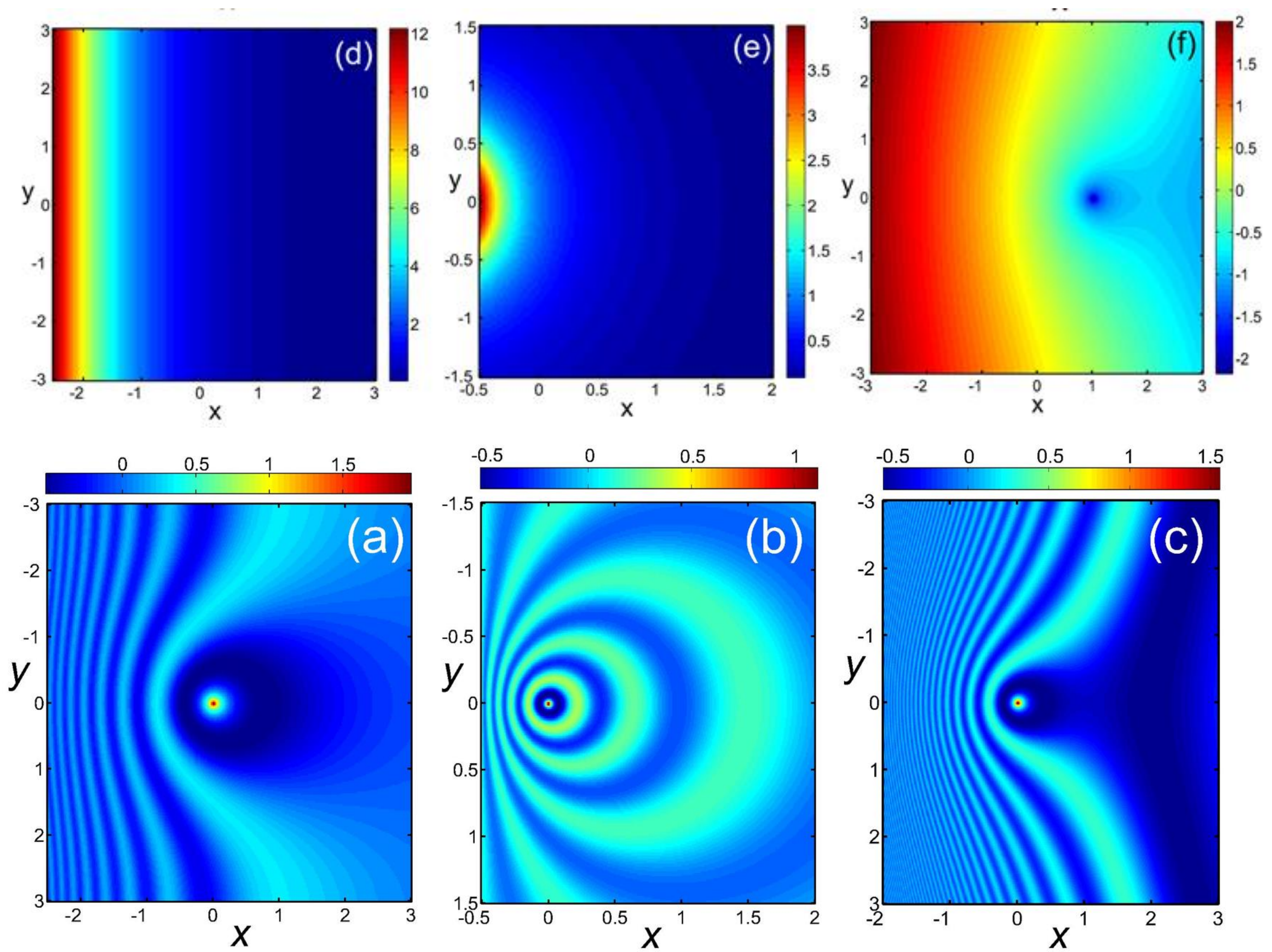
$$W = \frac{Z}{Z+1}$$

$$W = Ze^{-Z}$$



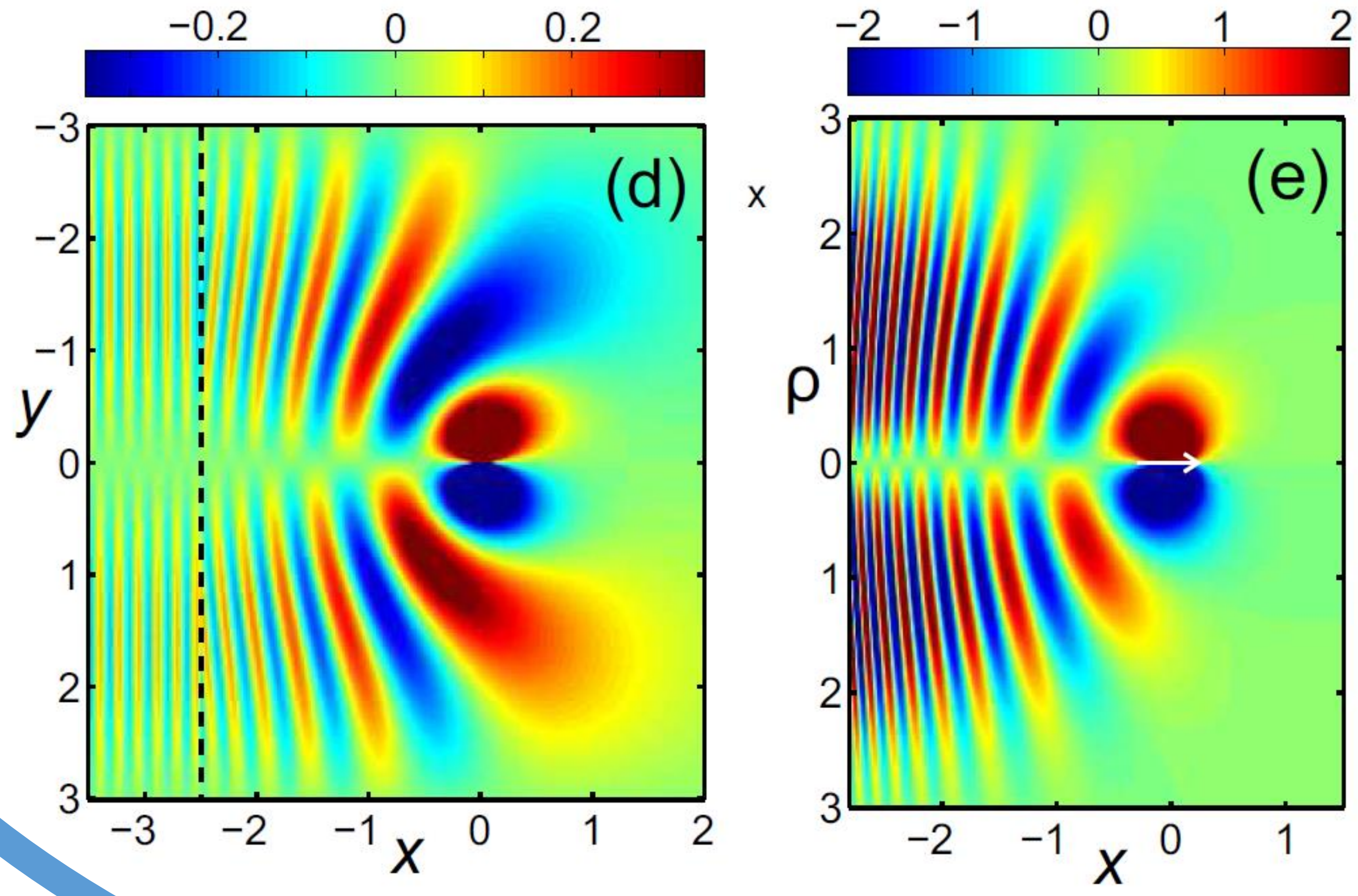
Monopole radiation

Perfectly collimated beams toward x → -∞



Going 3D

2D-Dipole in vs 3D-Dipole radiation in exponential medium

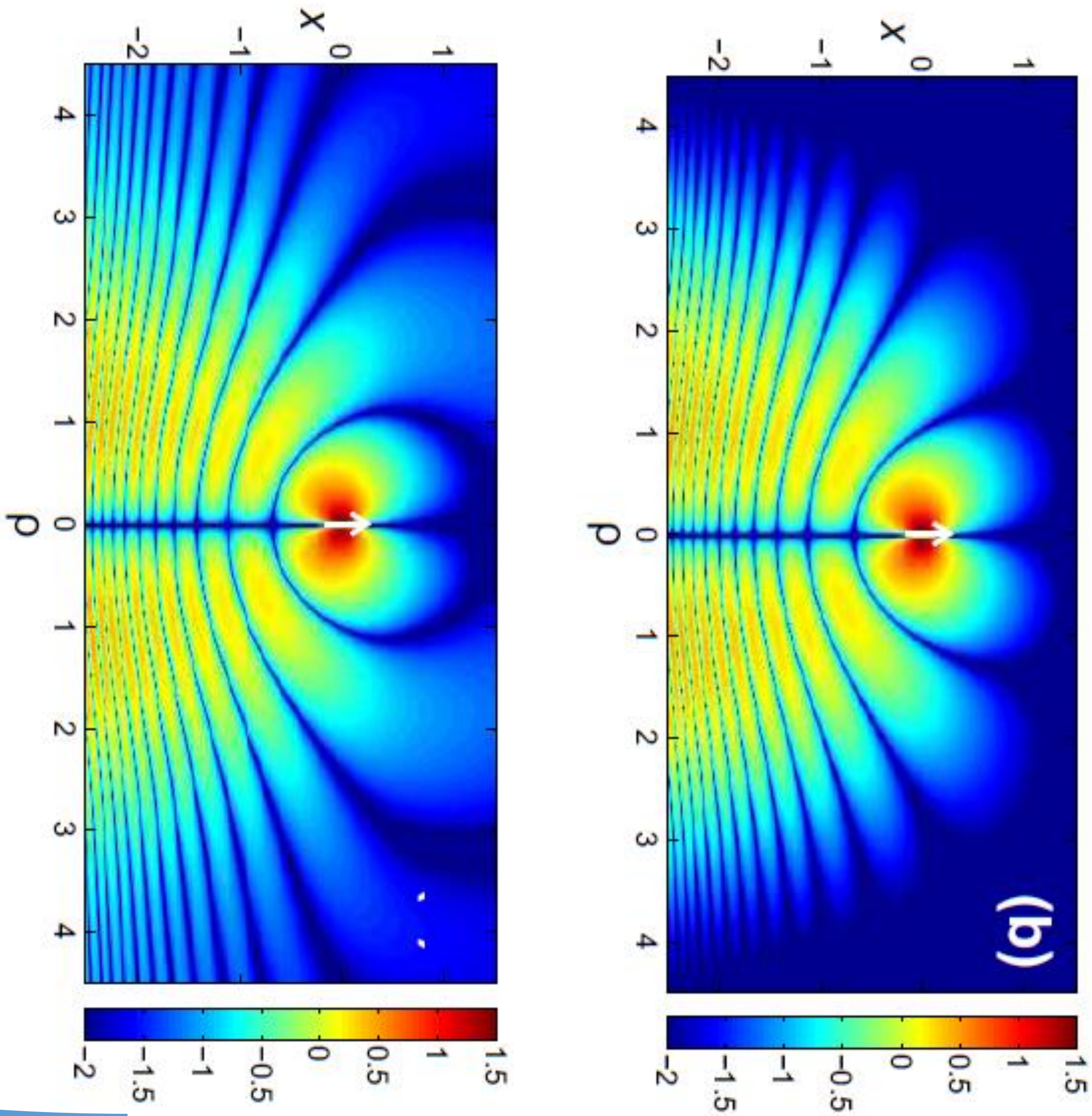


$$n(x) = e^{-x}$$

Going practical!

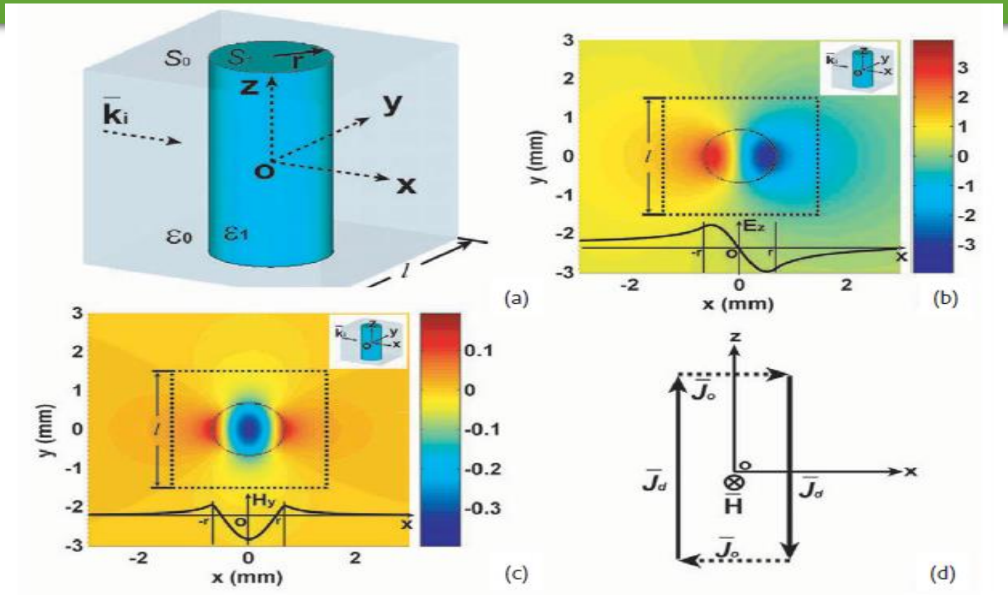
3D-dipole radiation in exponential medium (unbounded vs truncated on the low-index side)

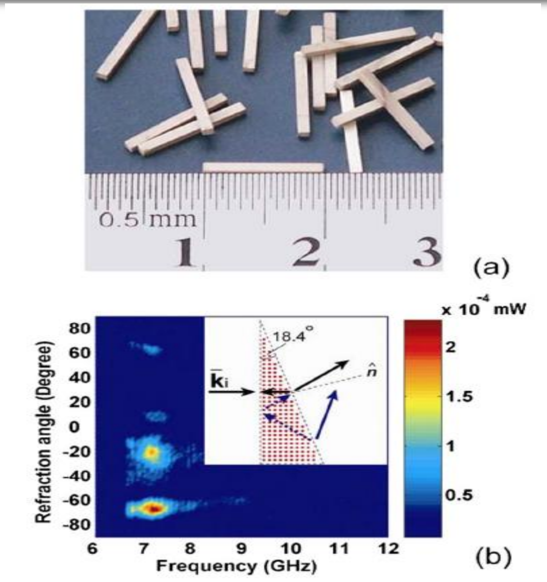
$n(x)$



All-Dielectric Metamaterials

Cylindrical Dielectric Resonators





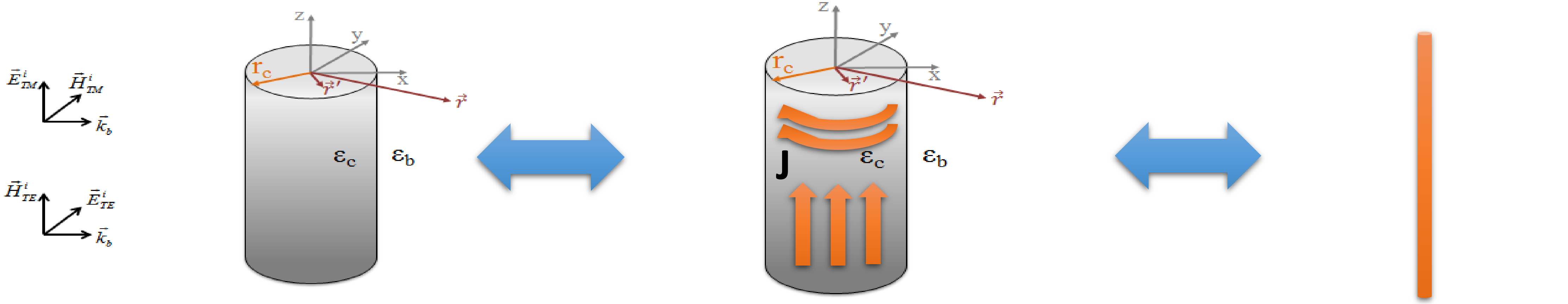
- Avoid plasmonic losses
- Physical principle: polarization currents
- Need high-ε materials (e.g. Si at optical frequencies)
- Anisotropic response
- → Retrieve effective medium parameters

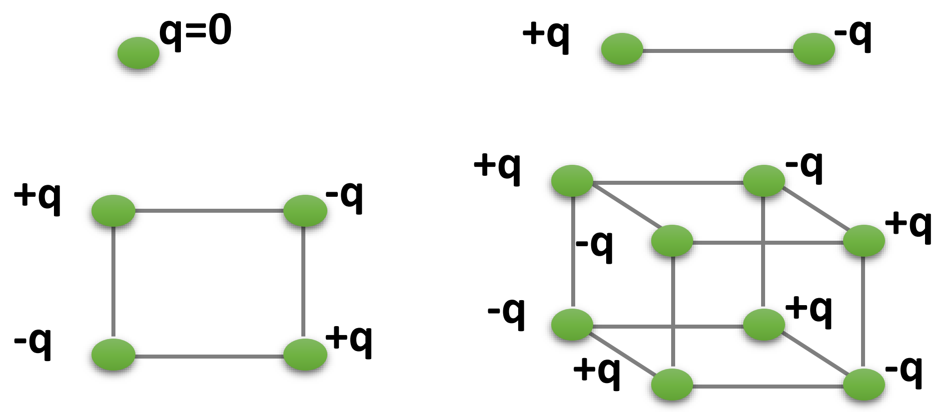
Peng et al., PRL (2007)

Isolated Resonators

Principle of Multipole Expansion

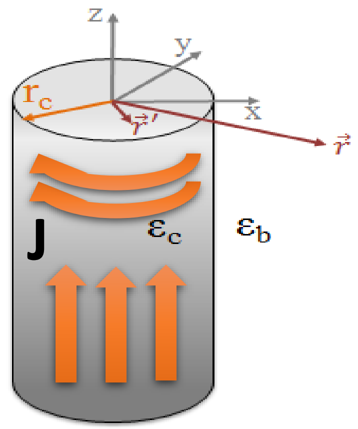
Incident Fields | Currents | Multipole Sources

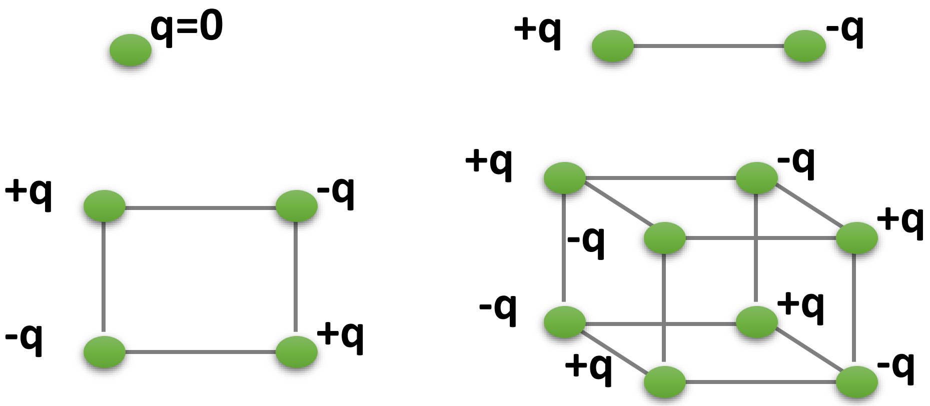




- Manipulate equivalent point-like elements
- Compatible with existing models (Maxwell-Garnett, etc.)

Multipole Expansion





$$\vec{A}(\vec{r}) = \mu_0 \int G(\vec{r}-\vec{r}\,')\vec{J}(\vec{r}\,')dS',$$

$$\varphi(\vec{r}) = \frac{1}{\varepsilon_0\varepsilon_b}\left[\int G(\vec{r}-\vec{r}\,')\rho(\vec{r}\,')dS' + \oint G(\vec{r}-\vec{r}\,')\sigma(\vec{r}\,')dl'\right]$$

$$multipoles \sim J \cdot r_c^n$$

$$\vec{p} = \frac{j}{\omega}\int_S \vec{J}(\vec{r}\,')dS'$$

$$\vec{m} = \frac{1}{2}\int_S \vec{r}\,' \times \vec{J}(\vec{r}\,')dS'$$

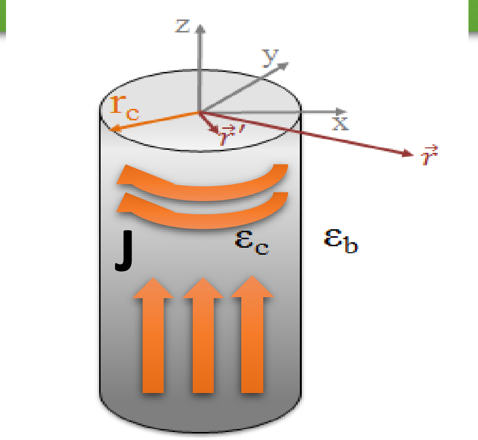
$$\overleftrightarrow{Q} = \frac{j}{\omega}\int_S \left[\vec{J} \otimes \vec{r}\,' + \vec{r}\,' \otimes \vec{J}\right]dS'$$

$$\vec{p} = \varepsilon_0\varepsilon_b \overleftrightarrow{\alpha}^e \cdot \vec{E}^i$$
$$\vec{m} = \overleftrightarrow{\alpha}^m \cdot \vec{H}^i$$
$$\overleftrightarrow{Q} = \varepsilon_0\varepsilon_b \overleftrightarrow{\alpha}^q \cdot \vec{E}^i$$

Multipole Expressions
Isolated Resonators

$$\vec{E}_{TM}/Z_b = +k_b^2\hat{z}\left[p_z c_b G + 2m_\varphi\, jG'\right]$$

$$\vec{H}_{TM} = +2k_b^2\left[\hat{n}m_n\left(G+G''\right)+\hat{\varphi}\, m_\varphi\left(G+\frac{G'}{k_b r}\right)\right]+k_b^2\hat{\varphi}\left[p_z c_b jG'\right]$$



$$\vec{E}_{TE}/Z_b = +k_b^2\left[\hat{n}p_n c_b\left(G+G''\right)+\hat{\varphi}\, p_\varphi c_b\left(G+\frac{G'}{k_b r}\right)\right]-k_b^2\hat{\varphi}\left[m_z jG'\right]$$

$$-\frac{k_b^2}{2}\left[\hat{n}\omega Q_{nn}\left(G'+G'''\right)+\hat{n}\omega Q_{\varphi\varphi}\left(\frac{G''}{k_b r}-\frac{G'}{\left(k_b r\right)^2}\right)+\hat{\varphi}\,\omega Q_{n\varphi}\left(G'-\frac{2G'}{\left(k_b r\right)^2}+\frac{2G''}{k_b r}\right)\right]$$

$$\vec{H}_{TE} = -k_b^2\hat{z}\left[p_\varphi c_b jG' - m_z G - \frac{j}{2}\omega Q_{n\varphi}\left(G''-\frac{G'}{k_b r}\right)\right]$$

$$\vec{E}_{3D}/Z = +k^3\left[\left(\hat{n}\cdot\vec{p}c\right)\hat{n}G''+\left(\vec{p}c\right)G+\left(\hat{n}\times\vec{m}\right)jG'\right]$$

$$\vec{H}_{3D} = -k^3\left[\left(\hat{n}\cdot\vec{m}\right)\hat{n}G''+\vec{m}G+\left(\hat{n}\times\vec{p}c\right)jG'\right]$$

Kallos et al., PRB (2012)

Fields – Multipoles Correlation

$$Multipoles \sim J \cdot r_c^n$$

$$\vec{E}/Z_b \overset{k_b r \gg 1}{\approx} +k_b^2\left[+\hat{\varphi}\left(p_\varphi c_b + m_z - \frac{j}{2}\omega Q_{n\varphi}\right) + \hat{z}\left(p_z c_b - 2m_\varphi\right)\right]G_\infty$$

$$\vec{H} \overset{k_b r \gg 1}{\approx} +k_b^2\left[+\hat{z}\left(p_\varphi c_b + m_z - \frac{j}{2}\omega Q_{n\varphi}\right) - \hat{\varphi}\left(p_z c_b - 2m_\varphi\right)\right]G_\infty$$

$$Mie \sim \left(r_c/\lambda_0\right)^n$$

$$\vec{E}_{TM} \overset{k_b r \gg 1}{\approx} -\hat{z}\left[b_0 + 2b_1\cos\varphi + 2b_2\cos 2\varphi\right]4jG_\infty$$

$$\vec{H}_{TE} \overset{k_b r \gg 1}{\approx} -\hat{z}\left[a_0 + 2a_1\cos\varphi + 2a_2\cos 2\varphi\right]4jG_\infty$$

$$\vec{p}_{TM} = +\hat{z}\frac{4b_0}{jk_b^2 Z_b c_b} \qquad \vec{m}_{TM} = -\hat{y}\frac{4b_1}{jk_b^2 Z_b} \qquad \overset{\leftrightarrow}{Q}_{TM} = \overset{\leftrightarrow}{0}$$

$$\vec{p}_{TE} = +\hat{y}\frac{8a_1}{jk_b^2 c_b} \qquad \vec{m}_{TE} = +\hat{z}\frac{4a_0}{jk_b^2} \qquad \overset{\leftrightarrow}{Q}_{TE} = \left(\hat{x}\hat{y} + \hat{y}\hat{x}\right)\frac{16a_2}{k_b^3 c_b}$$

$$\vec{p} = \varepsilon_0\varepsilon_b \overset{\leftrightarrow}{\alpha}^e \cdot \vec{E}^i$$

$$\vec{m} = \overset{\leftrightarrow}{\alpha}^m \cdot \vec{H}^i$$

$$\overset{\leftrightarrow}{Q} = \varepsilon_0\varepsilon_b \overset{\leftrightarrow}{\alpha}^q \cdot \vec{E}^i$$

Lattice of Resonators

Average Fields in Lattice

$$\vec{P}_{av}=\frac{\vec{p}}{S_{cell}}=\frac{1}{S_{cell}}\cdot\frac{j}{\omega}\int_{S_{cell}}\vec{J}\left(\vec{r}'\right)dS'$$

$$\vec{M}_{av}=\frac{\vec{m}}{S_{cell}}=\frac{1}{S_{cell}}\cdot\frac{1}{2}\int_{S_{cell}}\vec{r}'\times\vec{J}\left(\vec{r}'\right)dS'$$

$$\overleftrightarrow{Q}_{av}=\frac{\overleftrightarrow{Q}}{S_{cell}}=\frac{1}{S_{cell}}\cdot\frac{j}{\omega}\int_{S_{cell}}\left(\vec{J}\otimes\vec{r}'+\vec{r}'\otimes\vec{J}\right)dS'$$

$$\begin{pmatrix}\vec{E}_{av}\\ \vec{H}_{av}\end{pmatrix}=\frac{1}{S_{cell}}\int_{S_{cell}}\begin{pmatrix}\vec{e}\left(\vec{r}'\right)\\ \vec{h}\left(\vec{r}'\right)\end{pmatrix}dS'$$

Average Fields in Lattice

$$\vec{E}(\vec{r}) = \vec{e}(\vec{r})e^{j\vec{K}\cdot\vec{r}}, \quad \vec{H}(\vec{r}) = \vec{h}(\vec{r})e^{j\vec{K}\cdot\vec{r}},$$

$$\vec{K}\times\vec{E}_{av} = \omega\mu_0\vec{H}_{av}$$

$$\vec{K}\times\vec{H}_{av} \cong -\omega\varepsilon_0\varepsilon_b\vec{E}_{av} - \omega\vec{P}_{av} + \frac{j\omega}{2}\overleftrightarrow{Q}_{av}\cdot\vec{K} + \vec{K}\times\vec{M}_{av}$$

$$\vec{E}_{av} = \frac{\left(k_b^2\overleftrightarrow{I} - \vec{K}\vec{K}\right)}{\omega\varepsilon_0\varepsilon_b\left(k_b^2 - |\vec{K}|^2\right)}\cdot\left(-\omega\vec{P}_{av} + \frac{j\omega}{2}\overleftrightarrow{Q}_{av}\cdot\vec{K} + \vec{K}\times\vec{M}_{av}\right)$$

$$\vec{H}_{av} = \frac{\vec{K}}{k_b^2 - |\vec{K}|^2}\times\left(-\omega\vec{P}_{av} + \frac{j\omega}{2}\overleftrightarrow{Q}_{av}\cdot\vec{K} + \vec{K}\times\vec{M}_{av}\right)$$

From Maxwell's equations, we derive the fields averaged over a unit cell as function of the polarization current, or equivalently the multipole moments.

$$\vec{J}(\vec{r}) = -j\omega\varepsilon_0\left(\varepsilon(\vec{r}) - \varepsilon_b\right)\vec{E}(\vec{r})$$

$$\vec{K}\cdot\vec{r} = 1$$

Homogenization Process

$$\vec{K} \times \vec{E}_{eq} = \omega \vec{B}_{eq}$$
$$\vec{K} \times \vec{H}_{eq} \cong -\omega \vec{D}_{eq},$$

$$\vec{E}_{eq} = \vec{E}_{av} - \frac{j}{2\varepsilon_0 \varepsilon_b} \overleftrightarrow{Q}_{av} \cdot \vec{K}$$
$$\vec{B}_{eq} = \mu_0 \vec{H}_{av} - \frac{j}{2\omega \varepsilon_0 \varepsilon_b} \vec{K} \times \overleftrightarrow{Q}_{av} \cdot \vec{K}$$
$$\vec{H}_{eq} = \vec{H}_{av} - \vec{M}_{av}$$
$$\vec{D}_{eq} = \varepsilon_0 \varepsilon_b \vec{E}_{av} + \vec{P}_{av} - \frac{j}{2} \overleftrightarrow{Q}_{av} \cdot \vec{K}$$

$$\vec{D}_{eq} = \varepsilon_0 \overleftrightarrow{\varepsilon}_{eff} \cdot \vec{E}_{eq}$$
$$\vec{B}_{eq} = \mu_0 \overleftrightarrow{\mu}_{eff} \cdot \vec{H}_{eq}$$

We define an equivalent medium without any current present

While the original lattice in non-magnetic, the equivalent medium is

The system can be viewed either as

1) the original non-magnetic medium with a magnetic response arising from the supported electric polarization currents only, or
2) as the equivalent magnetic medium with the magnetic response arising from the effective permeability

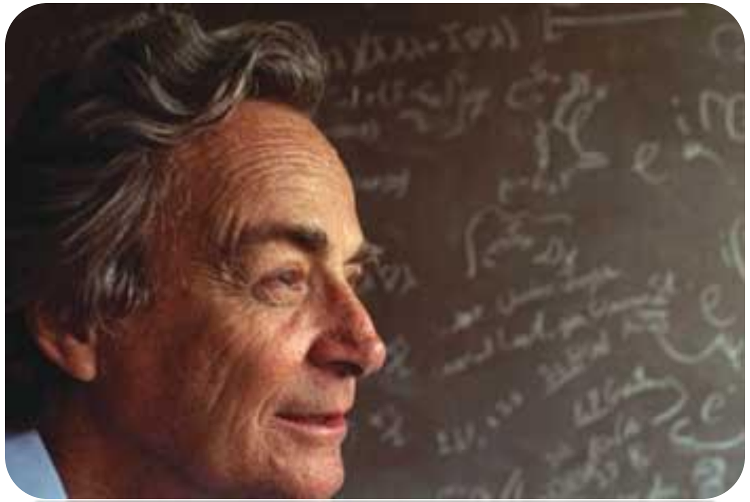
Effective Parameters

$$\frac{\varepsilon_{eff}^{zz}}{\varepsilon_b} = 1 + \left(\left| \frac{\vec{K}}{k_b} \right|^2 - 1 \right) \frac{P_{z,av} k_b c_b}{P_{z,av} k_b c_b - 2\hat{z} \cdot \left(\vec{K} \times \vec{M}_{av} \right)}$$

$$\mu_{eff}^{zz} = 1 + \left(\left| \frac{\vec{K}}{k_b} \right|^2 - 1 \right) \frac{k_b^2 M_{z,av} - (j\omega / 2)\hat{z} \cdot \left[\vec{K} \times \left(\overleftrightarrow{Q}_{av} \cdot \vec{K} \right) \right]}{k_b^2 M_{z,av} + \hat{z} \cdot \left(\vec{K} \times \vec{P}_{av} k_b c_b \right) - (j\omega / 2)\hat{z} \cdot \left[\vec{K} \times \left(\overleftrightarrow{Q}_{av} \cdot \vec{K} \right) \right]}$$

Conclusions

- *Practical, all-dielectric, isotropic* transformation media are proposed to be used for perfectly mode-matched interaction of light with single quantum emitters.
- The designs are based on *conformal mappings* that parallelize radial rays, in the strict or asymptotic sense, and on their revolution-symmetric 3D counterparts.
- The proposed media are *non-resonant* and straightforward to implement as smooth refractive index landscapes on a monolithic dielectric platform with existing techniques.



“I can’t see what exactly would happen,

but I can hardly doubt that when we have some control of the arrangement of things in the small scale,

we will get an enormously greater range of possible properties that substances can have.”

1959

R. Feynman, There's Plenty of Room at the Bottom
http://www.zyvex.com/nanotech/feynman.html

Thank You!

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